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THESIS

INHERENT ANGULAR TRACKING ERROR
IN AN AMPLITUDE COMPARISON MONOPULSE RADAR

by

Dae Hyun Park

December 1984

Thesis Advisor:

Hung-Mou Lee

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Inherent Angular Tracking Error
in an Amplitude Comparison Monopulse Radar

by

Dae Hyun Park
Major, Republic of Korea Army
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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1984

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Amplitude comparison monopulse tracking of a two-element target is studied. A Gaussian radiation pattern for the antennas is assumed. It is demonstrated that the cross over angle of the antenna assembly and the target angular span are essential parameters for determining the angular tracking error. Computational results showing the inherent error curve, $X_{\alpha}(A)$, are provided.

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I. INTRODUCTION

A. TRACKING RADAR

A tracking radar is used to determine the range and angular location of a target. Three angle tracking techniques are commonly employed. They are conical scanning, sequential lobing and monopulse. A conical scan radar requires at least three returned pulses in one full scan to determine the target angular location.

If the amplitude of the target echo changes appreciably during this three-pulse period, there will be tracking errors even if the target is an ideal point target. More specifically, the target echo power arriving at the radar antenna changes from time to time. This fluctuation in echo signal power at the radar antenna will be called target scintillation, which includes amplitude scintillation and phase scintillation. The angular tracking noise, which is the deviation of the estimated target location by the radar from the actual target angular location, will contain an amplitude modulation at the scanning frequency due to the amplitude scintillation of the target. That is, the tracking noise in conical scan radar is a function of the spectral density of the amplitude scintillation. Since target scintillation is due mostly to the motion of the target, its spectral density falls off at high frequencies. It is desirable to raise the scanning rate as high as achievable with mechanically moving parts. This is one of the major reasons for the development of the sequential lobing radars which can be scanned electronically. The limit of the lobing rate is the pulse repetition rate, which is usually not achievable with a conically scanning radar.

A monopulse radar obtains, with one pulse, a complete elevation difference signal, an azimuth difference signal and a sum signal to estimate target position. Since target scintillation within a pulse duration is usually negligible, it will not cause tracking errors in a monopulse radar.

B. NOISE AND TRACKING ERROR

According to Howard et.al. [Ref. 1], the sources of noise which cause tracking errors can be classified into four major categories. They are servo noise, receiver noise, angle noise and amplitude noise.

Servo noise is generated in the hunting action of the tracking servo mechanism which results from backlash and compliance in the gears, shafts, and structures of the mount. The magnitude of this noise is essentially independent of the target and will thus be independent of range.

Receiver noise is the effect on the tracking accuracy of the radar due to thermal noise generated in the receiver and any spurious hum which may be picked up by the circuitry.

Angle noise is the tracking error introduced into the radar by variations in the apparent angle of arrival of the echo from a complex target of finite size. This effect is caused by variations in the phase front of the radiation from an extended target as the target changes its aspect. The magnitude of angle noise is inversely proportional to the range of the target.

Amplitude noise is the effect on the radar accuracy due to the fluctuations in the amplitude of the signal returned by the target. These fluctuations are caused by any change in aspect of the target and must be taken to include propeller rotation and skin vibration. Figure 1.1, adopted from Howard et.al. [Ref. 1], shows the relative amplitudes of these noise components versus relative range. For a CONSCAN radar (A) and monopulse radar (B).

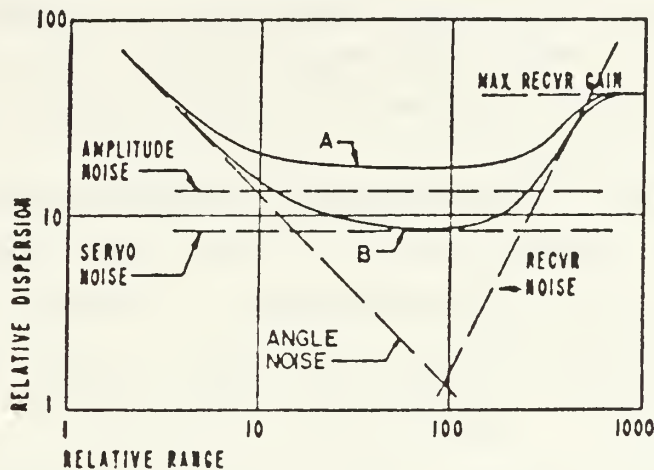


Figure 1.1 Relative Dispersion Caused by Radar Noise Components from Howard.

It can be seen immediately that the above classification by Howard is inadequate because each category includes only time-dependent noise. Servo noise and receiver noise are dependent only on the radar while angle noise and amplitude noise are dependent only on the target motion. Hence there should be no angle noise and amplitude noise in the estimated target angular coordinates with a monopulse radar. Howard in [Ref. 1] and [Ref. 2] pointed out that the distortion of the target phase front itself, not the time change of the phase front, would introduce errors into the radar.

He then claimed that all radars, including amplitude comparison monopulse radars and search radars, are affected by the phase front distortion in the same manner.

Although he never made it clear what he meant by phase front distortion, his definition about angle error was inconsistent, and his claims are false except when applied to phase comparison monopulse radars, he nevertheless recognized the fact that it was not only the change in target echo which introduced noise into the radar tracking direction but the radar system might respond to a particular target echo by pointing its tracking axis off the target.

It is clear that the angular tracking error of a radar is not just caused by noise. It sets in through the designed target locating mechanism. The radar will respond to the echo from a target by pointing its tracking axis to a particular direction which is usually called the apparent angle of arrival. The response function which translates the echo signal received at the antenna assembly into an angular tracking error is not random. Rather, it is a deterministic result of the radar system design. A tracking system is designed to track a point target. When the target is actually an extended scatterer, the radar system will respond erroneously.

The angular error in this response will be called the inherent angular tracking error and the rule which determines this response for a monopulse tracking radar is time-independent. Thus the four categories of noise which cause tracking errors as defined by Howard are inappropriate because only time-dependent sources are included. His inclusion of the phase front distortion is inadequate because both amplitude and phase distortions of the target echo will cause angular tracking errors in a tracking radar. His phase front distortion technique is an approximation with limited validity.

A noise source should be classified according to whether it is target dependent or target independent. A target dependent tracking noise which is the result of target

scintillation, in both amplitude and phase, transformed through a radar system dependent (and target independent) mechanism. Because of the transformation by the radar system, the tracking noise need not be the same as the target scintillation, and will not be the same in different radars which are tracking the same target. There have been studies of target scintillation. A careful study of the inherent angular tracking error of a phase comparison monopulse tracking system has been carried out by Lee [Ref. 3]. For an amplitude comparison monopulse system, it appears that both amplitude scintillation and phase scintillation of the target echo cause tracking errors because of the target tracking mechanism of the radar system [Ref. 4], [Ref. 5].

C. PHASE FRONT DISTORTION AND TRACKING ERROR OF A PHASE COMPARISON MONOPULSE TRACKING SYSTEM

A point target will radiate an echo signal which is spherically symmetric in the far field. A tracking radar is designed to track such a target accurately. An extended target will radiate an echo signal, in response to the incident wave from a radar, which is not spherically symmetric in either amplitude or phase. A phase comparison tracking radar is constrained to have individual feeds in its antenna assembly at locations which have the same phase, modulo 2π .

There are two types of angular tracking errors: local and global. If a phase front is defined to be a surface on which the target echo has constant phase, then the radar has a local error if the feeds determine the same angular coordinate are on the same phase front. The radar has a global error if at least one of its feeds determine one angular coordinate (elevation or azimuth) is on different phase fronts.

A local error is present because the antennas have to adjust their radial distances from the target so as to compensate the differences in phase of the echo signals arrive at different antennas. Howard's technique in determining this error angle approximates this phase compensation mechanism for antennas of finite separation with the phase front gradient at the center of the antennas and thus exaggerates the error. The existence of the global error which will lead the radar to its eventual loss of track is a new feature discovered by Lee [Ref. 3]. These recent results pointed out the importance of studying the inherent angular tracking error as a radar system characteristic.

D. THE PURPOSE OF THIS THESIS

Meade [Ref. 6] initiated the study of the angular tracking error caused by a two reflector target. He assumed an amplitude comparison system and looked into the effect on tracking accuracy due to the finite angular extent of the target over the antenna lobe. He cautioned that his result would not apply if the angular tracking error should become large because only two terms in a Taylor series expansion of the lobe pattern in the direction of the tracking axis are included in his analysis. Lee [Ref. 7] pointed out the fact that when the next higher term in the Taylor series expansion is added, a qualitatively different result is obtained. The purpose of this thesis is to analyze this problem in greater detail by utilizing a Gaussian beam pattern for the antenna. Tracking errors up to the order of the antenna beam width can thus be discussed.

II. FORMULATION OF THE PROBLEM

A. AMPLITUDE COMPARISON MONOPULSE TRACKING RADAR

In the monopulse tracking radar, a complex target has been always treated as a point target in electromagnetic scattering theory dealing with the far field. When monopulse tracking is concerned, this is good assumption only if the target angular span is negligible compared to the beam width of the antenna. There is no angular error in amplitude comparison monopulse tracking of a point target. The design of the monopulse tracking radar is based on the assumption that the target being tracked is a point target. But when a target spans an angle which is not negligible compared to the beam width, the tracking error analysis becomes a complicated problem. The angular span relative to the antenna beam width is a major parameter in the determination of inherent angular tracking error of an amplitude comparison monopulse radar.

B. A TARGET HAVING TWO INDEPENDENT SCATTERERS

To analyze the angular tracking error of an amplitude comparison monopulse radar, it is assumed that the target has two independent scatterers. And for a fixed azimuth angle or a fixed elevation angle, only a two-dimensional situation will be considered and only two antennas are needed. A Gaussian beam pattern is assumed so that the tracking error up to the order of the beam width of the antenna can be analyzed. Angular tracking error as a function of the phase difference and amplitude ratio of the two independent scatterers will be studied. The formulation below follows that of Lee [Ref. 4].

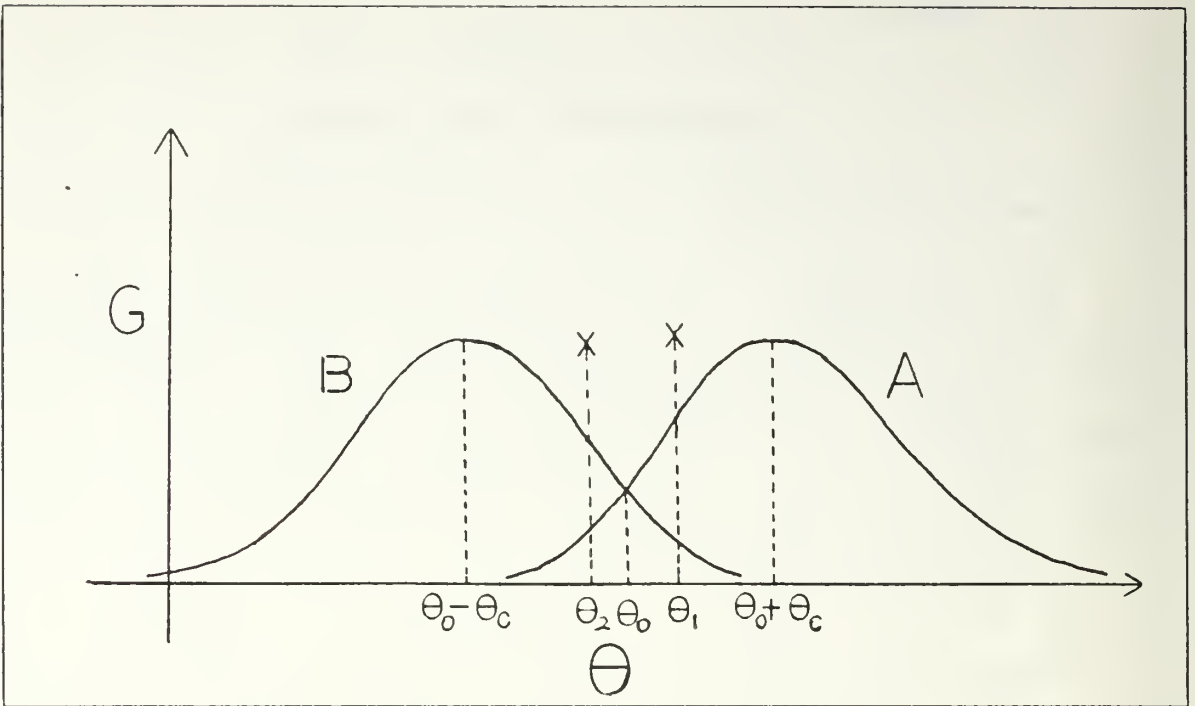


Figure 2.1 Azimuth Plane in Amplitude Comparison Monopulse Tracking System.

The two antennas A and B have identical Gaussian radiation patterns of constant phases, with the half power beam-width Θ_B . The antenna beam axes are offset to opposite sides from the tracking axis by the cross over angle Θ_c . The boresight axis points along the direction Θ_0 . The angular locations of the two scatterers are Θ_1 and Θ_2 respectively, with $\Theta_1 > \Theta_2$. The target center Θ_T is the angular center of the two scatterers,

$$\Theta_T = \frac{\Theta_1 + \Theta_2}{2}$$

The target angular span is;

$$\begin{aligned}\theta_s &= \frac{\theta_1 - \theta_2}{2} \\ &= \theta_i - \theta_T \\ &= \theta_T - \theta_2 > 0.\end{aligned}$$

The angular tracking error is measured from the target center to the direction of the tracking axis.

$$\theta_x = \theta_o - \theta_T$$

The lobe pattern of antenna A is:

$$G_A(\theta) = \frac{-2}{2} \left(\frac{\theta - \theta_o - \theta_c}{\theta_B} \right)^2 \quad (2-1)$$

The lobe pattern of antenna B is:

$$G_B(\theta) = \frac{-2}{2} \left(\frac{\theta - \theta_o + \theta_c}{\theta_B} \right)^2 \quad (2-2)$$

Assume that the ratio of the scattered field strength of two scatterers to be $2^A e^{j\alpha}$.

The lobe voltages of the received signals by the antennas are proportional to:

$$|V_A| = \left| 2^A e^{j\alpha} 2^{-2\left(\frac{\theta_1 - \theta_0 - \theta_c}{\theta_B}\right)^2} + 2^{-2\left(\frac{\theta_2 - \theta_0 - \theta_c}{\theta_B}\right)^2} \right| \quad (2-3)$$

$$|V_B| = \left| 2^A e^{j\alpha} 2^{-2\left(\frac{\theta_1 - \theta_0 + \theta_c}{\theta_B}\right)^2} + 2^{-2\left(\frac{\theta_2 - \theta_0 + \theta_c}{\theta_B}\right)^2} \right| \quad (2-4)$$

Define the new variables which are normalized to the half-power beamwidth.

$$X_c = \frac{2\theta_c}{\theta_B} \quad ; \text{ normalized beam separation.}$$

$$X_s = \frac{2\theta_s}{\theta_B} \quad ; \text{ normalized target span.}$$

$$X = \frac{2\theta_x}{\theta_B} \quad ; \text{ normalized angular tracking error.}$$

The expressions for $|V|$ and $|V|$ may now be simplified. In terms of X_c , X_s and X ,

$$|V_A|^2 = 2^{A-(X^2+X_c^2+X_s^2)-2XX_c} \cdot \left[\right] \quad (2-5)$$

where, $\left[\right] = 2^{A+2X_s(X+X_c)} + 2^{-A-2X_s(X+X_c)} + 2\cos(\alpha)$

$$|V_A|^2 = 2^{A-(X^2+X_c^2+X_s^2)-2XX_c} \cdot \left[\right] \quad (2-6)$$

where, $\left[\right] = 2^{A+2X_s(X-X_c)} + 2^{-A-2X_s(X-X_c)} + 2\cos(\alpha)$

From the error signal nulling condition, $|V|^2 - |V|^2 = 0$, the following formula is obtained:

$$2^{4XX_c} = \frac{2^{A+2X_s(X+X_c)} + 2^{-A-2X_s(X+X_c)} + 2\cos(\alpha)}{2^{A+2X_s(X-X_c)} + 2^{-A-2X_s(X-X_c)} + 2\cos(\alpha)} \quad (2-7)$$

This equation determines the angular tracking error when an amplitude comparison monopulse radar is used to track a two-element target. When the target angular span is negligible compared to the beam width, that is, when X_s approaches 0, $X = 0$ is a solution of the above equation.

III. THE TRACKING ERROR CURVES

The tracking error of an amplitude comparison monopulse radar is analyzed. Error curves are obtained through the use of a digital computer. Equation 2.7 can be split into two parts, each forming a three dimensional surface:

$$Z_1 = 2^{A+2X_s X_c} \quad (3-1)$$

$$Z_2 = \frac{2^{A+2X_s(X+X_c)} + 2^{-A-2X_s(X+X_c)} + 2\cos(\alpha)}{2^{A+2X_s(X-X_c)} + 2^{-A-2X_s(X-X_c)} + 2\cos(\alpha)} \quad (3-2)$$

A curve is obtained on the A-x plane as the projection of the intersection of the two surfaces given by equations (3.1) and (3.2). Coordinate transformation to the variable $A+2XX_s$ is carried out for equation (3.2) to facilitate numerical computation by a digital computer. The solution for the normalized angular tracking error can be represented as a curve in the A-X plane with the phase difference between the echo signals from the scatterers as the parameter. Consider a target which has two scattering centers separated by one meter and assume the target is 40 km away from a tracking radar. Assume the antennas of the radar have a half-power beamwidth of 1.5 degrees, then $X_s = 0.001$ degrees. The normalized angular tracking errors under this assumption are studied.

A. CASE 1: $X_C=0.3$, $X_S=0.001$ FOR SEVERAL PHASE DIFFERENCES

Figure 3.1 through 3.11 show X , the normalized tracking error, versus A with phase difference between the scattered field from the two scatterers as the parameter.

Note that 2^A is the ratio between the strength of the scattered fields.

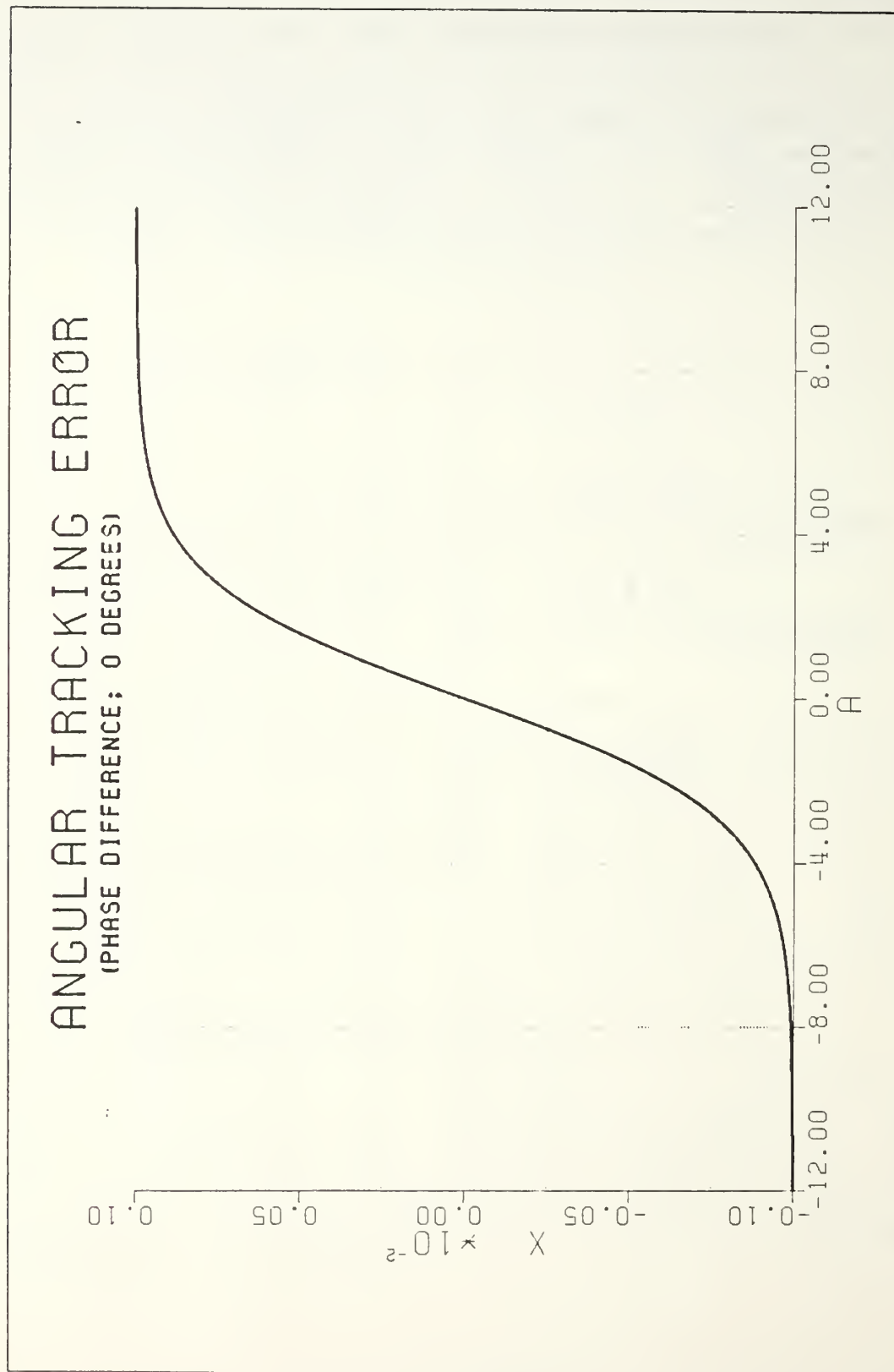


Figure 3.1 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 0 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 90 DEGREES)

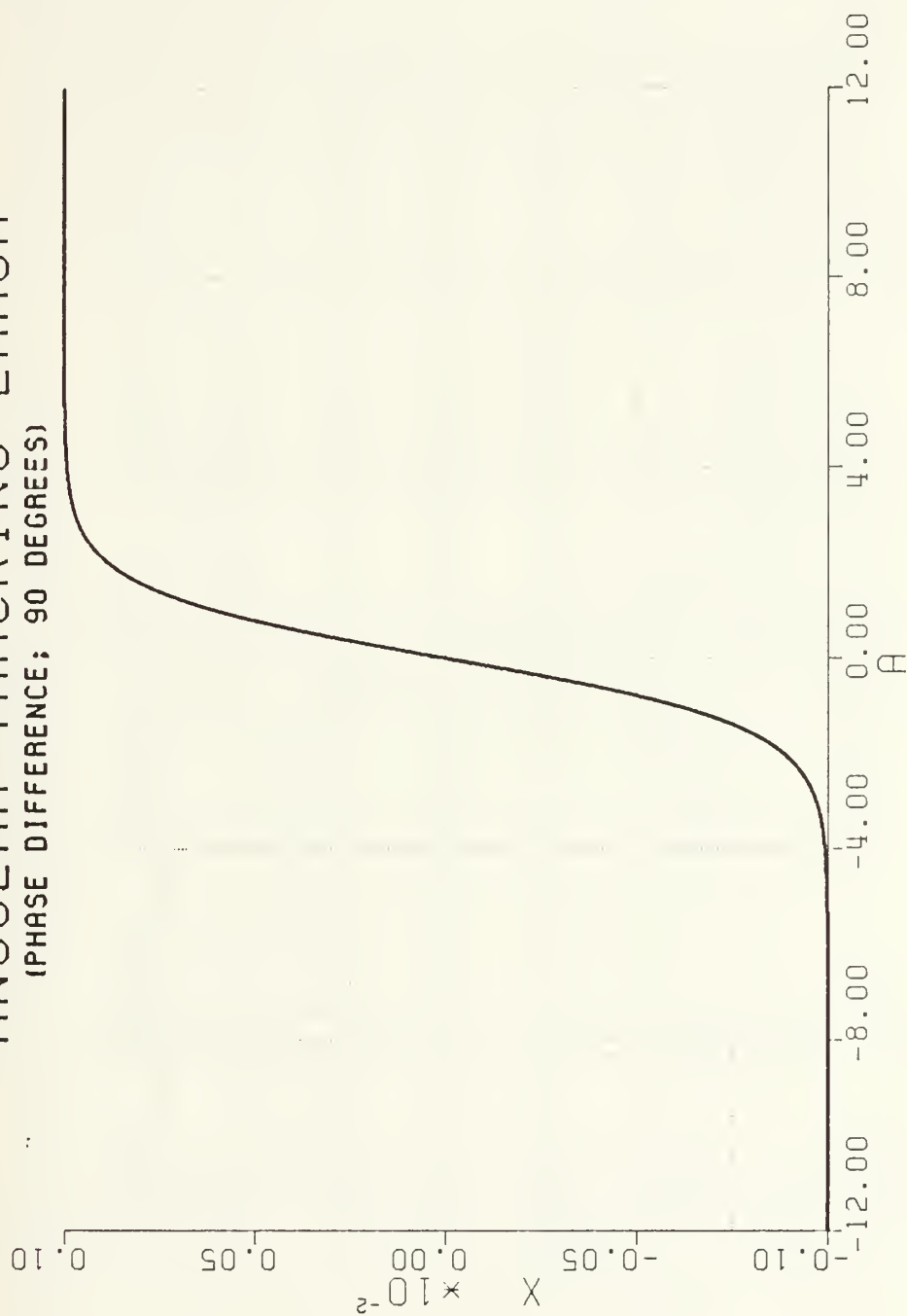


Figure 3.2 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 100 DEGREES)

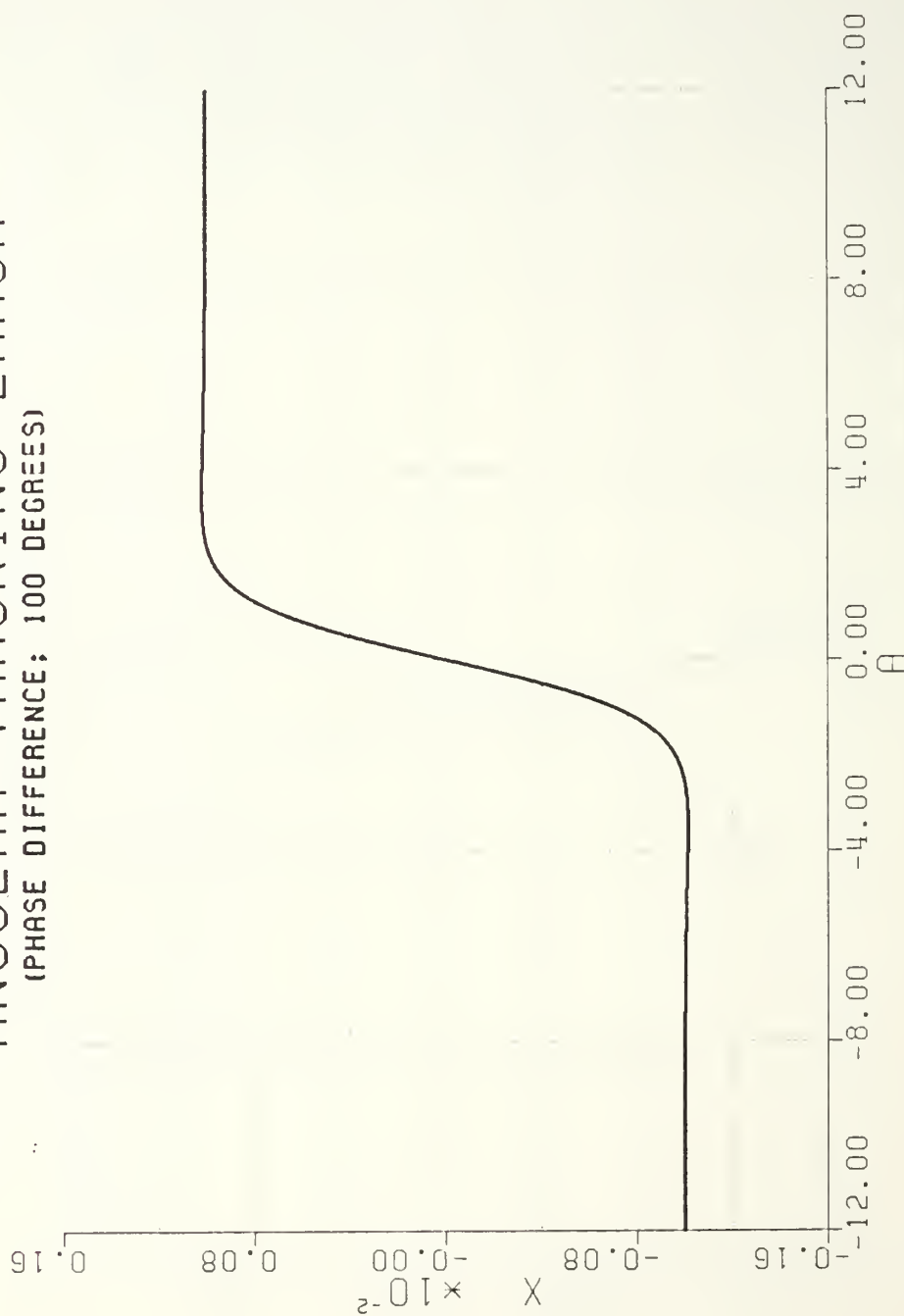


Figure 3.3 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 100 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 105 DEGREES)

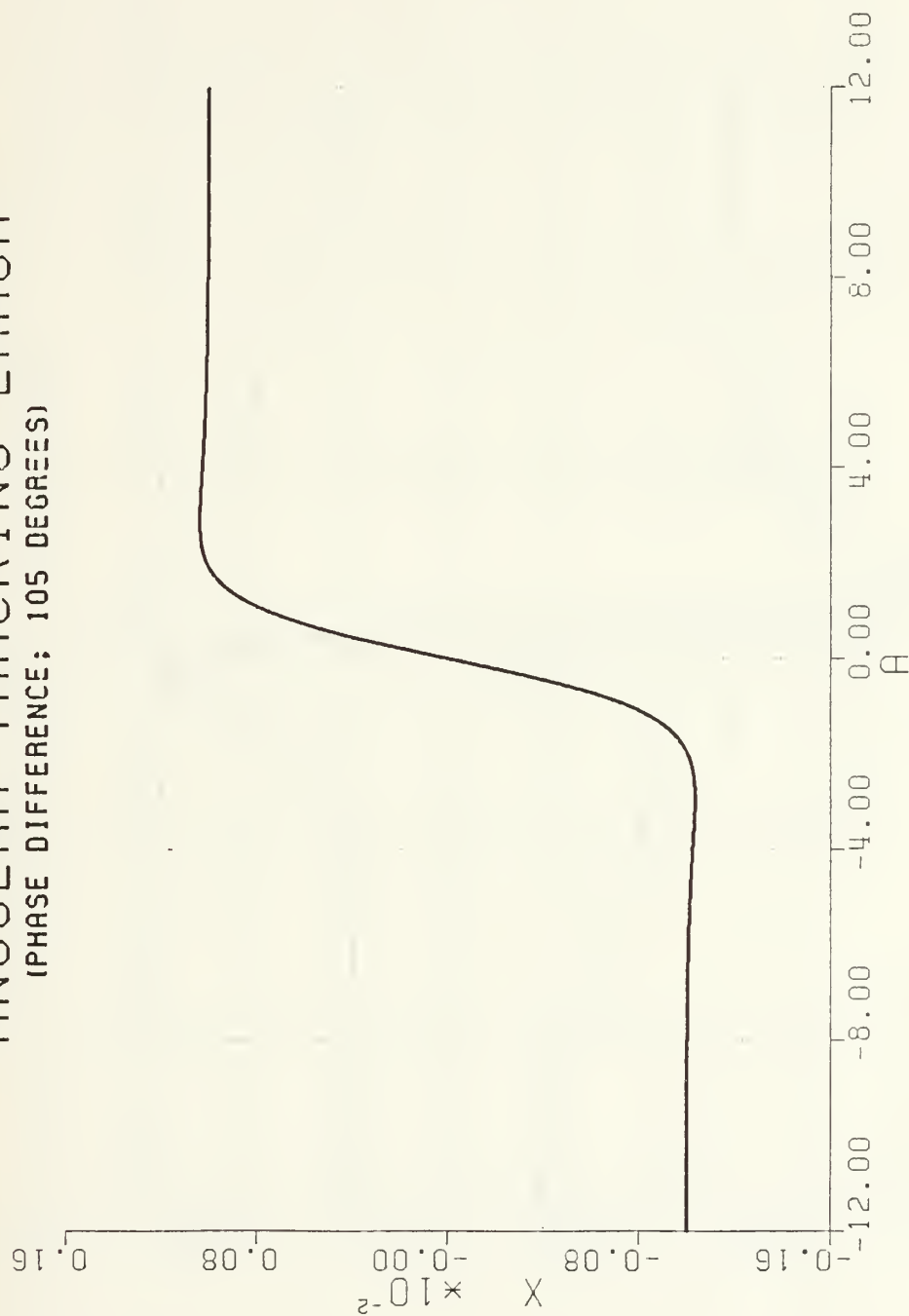


Figure 3.4 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 105 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 150 DEGREES)

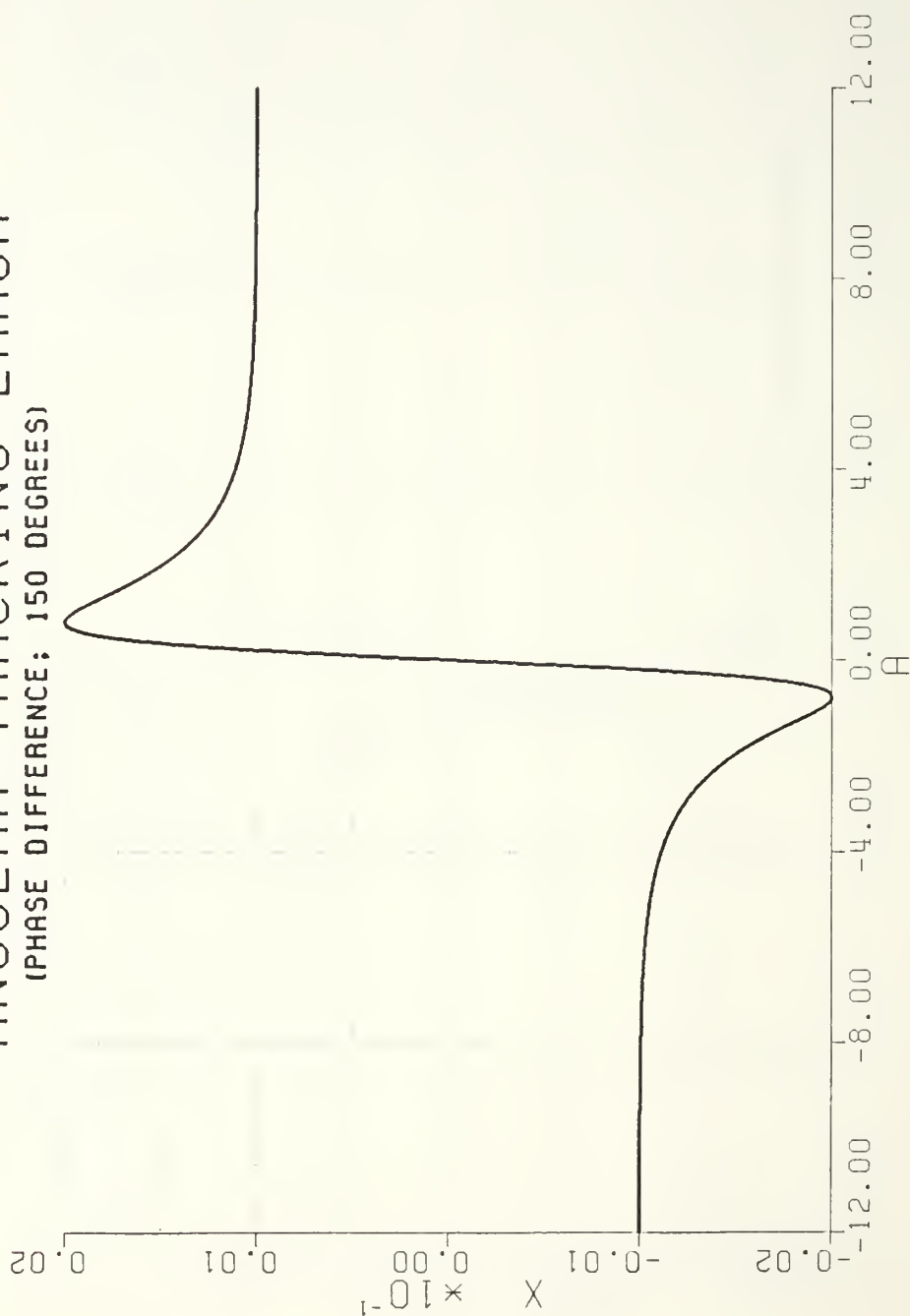


Figure 3.5 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 150 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.88 DEGREES)

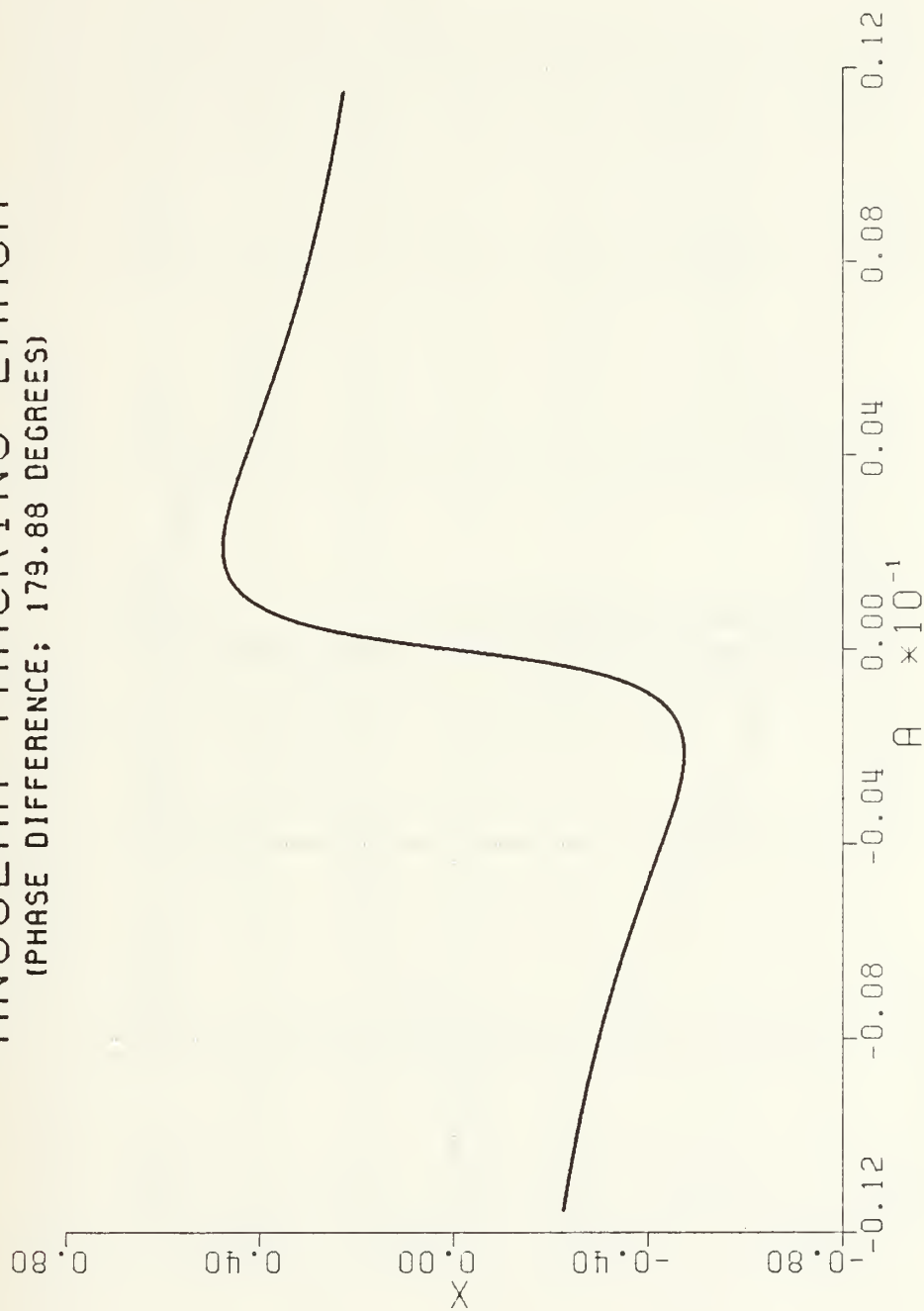


Figure 3.6 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.88 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.90 DEGREES)

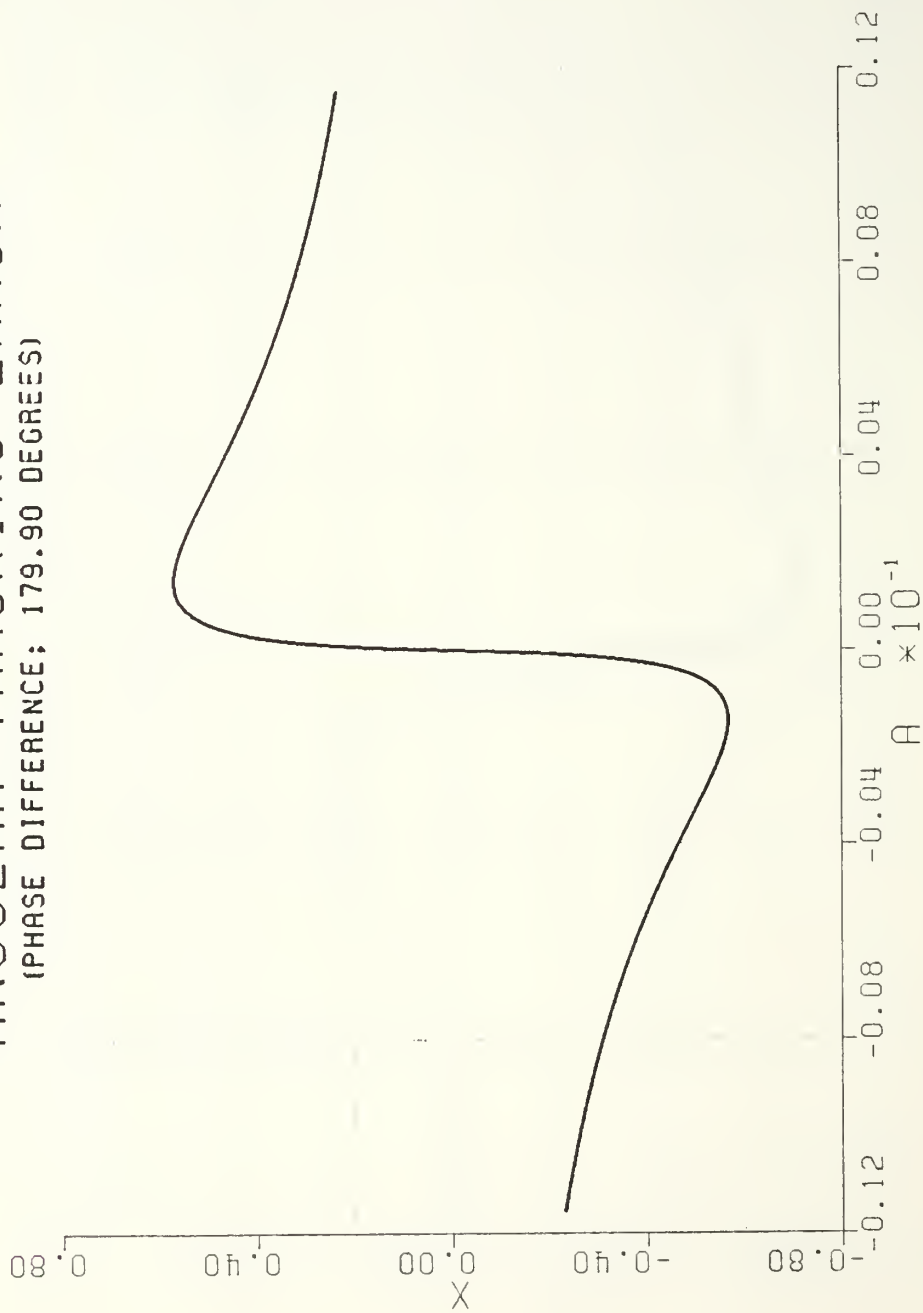


Figure 3.7 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.91 DEGREES)

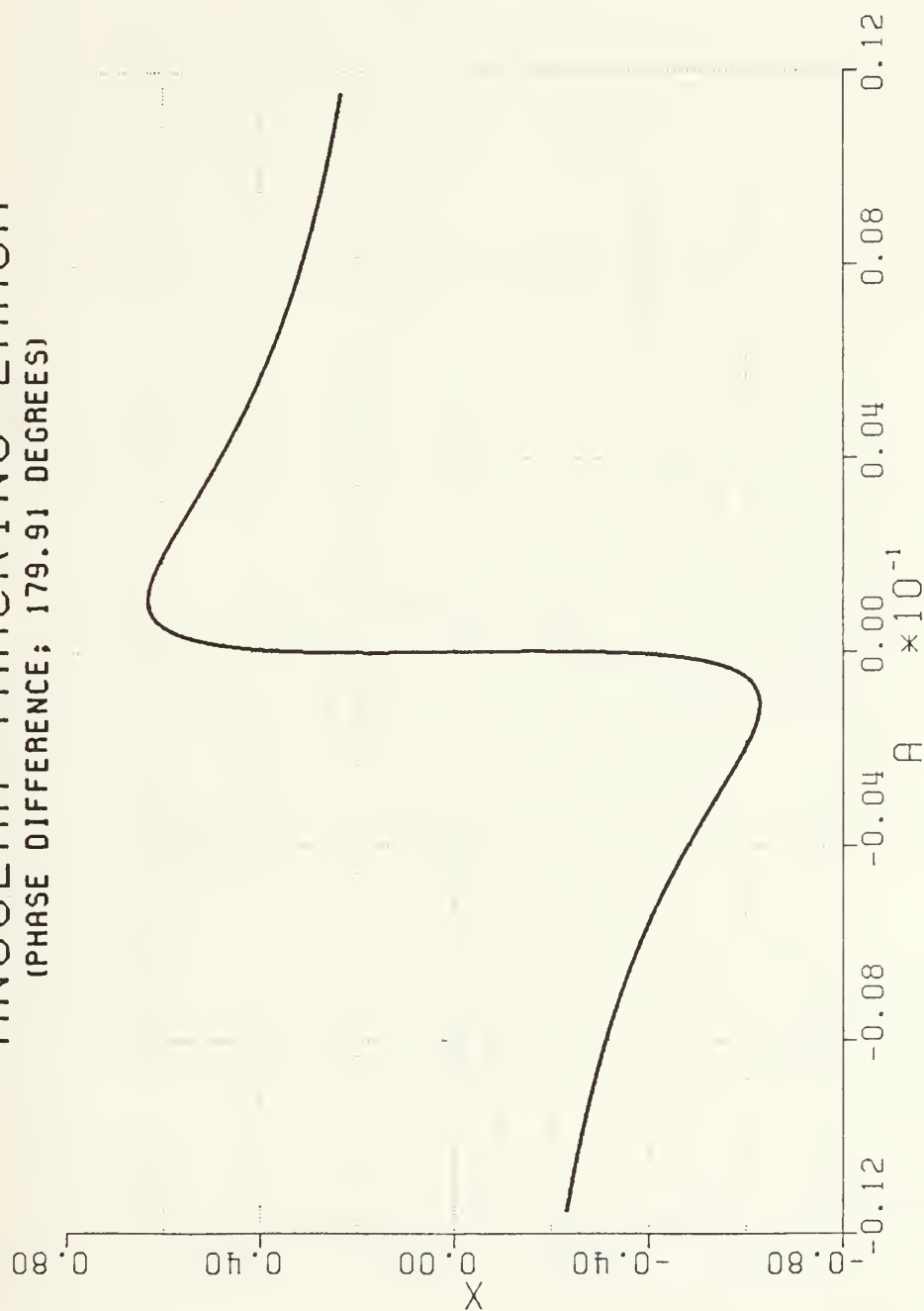


Figure 3.8 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.91 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.95 DEGREES)

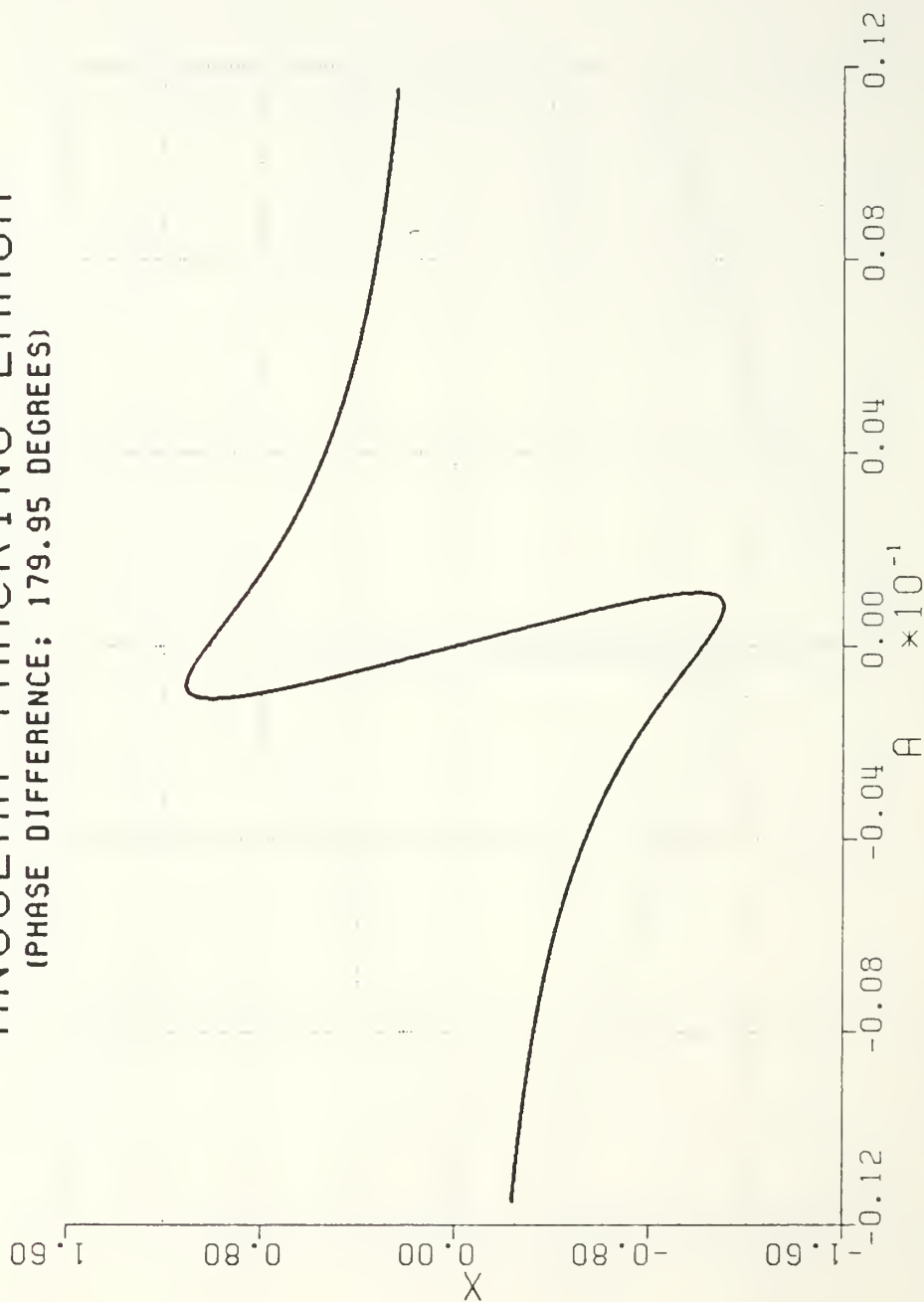


Figure 3.9 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.95 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.99 DEGREES)

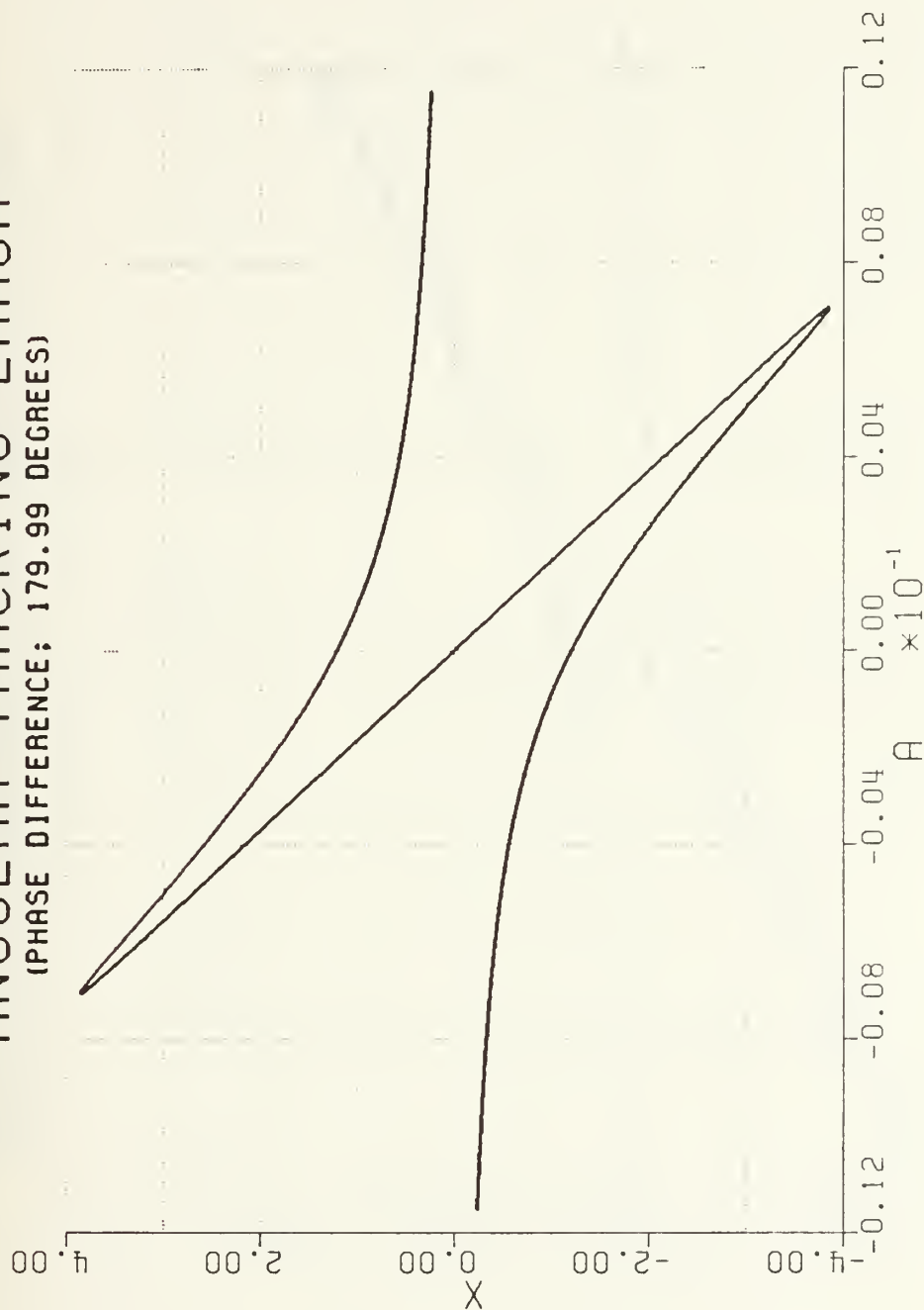


Figure 3.10 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.99 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 180 DEGREES)

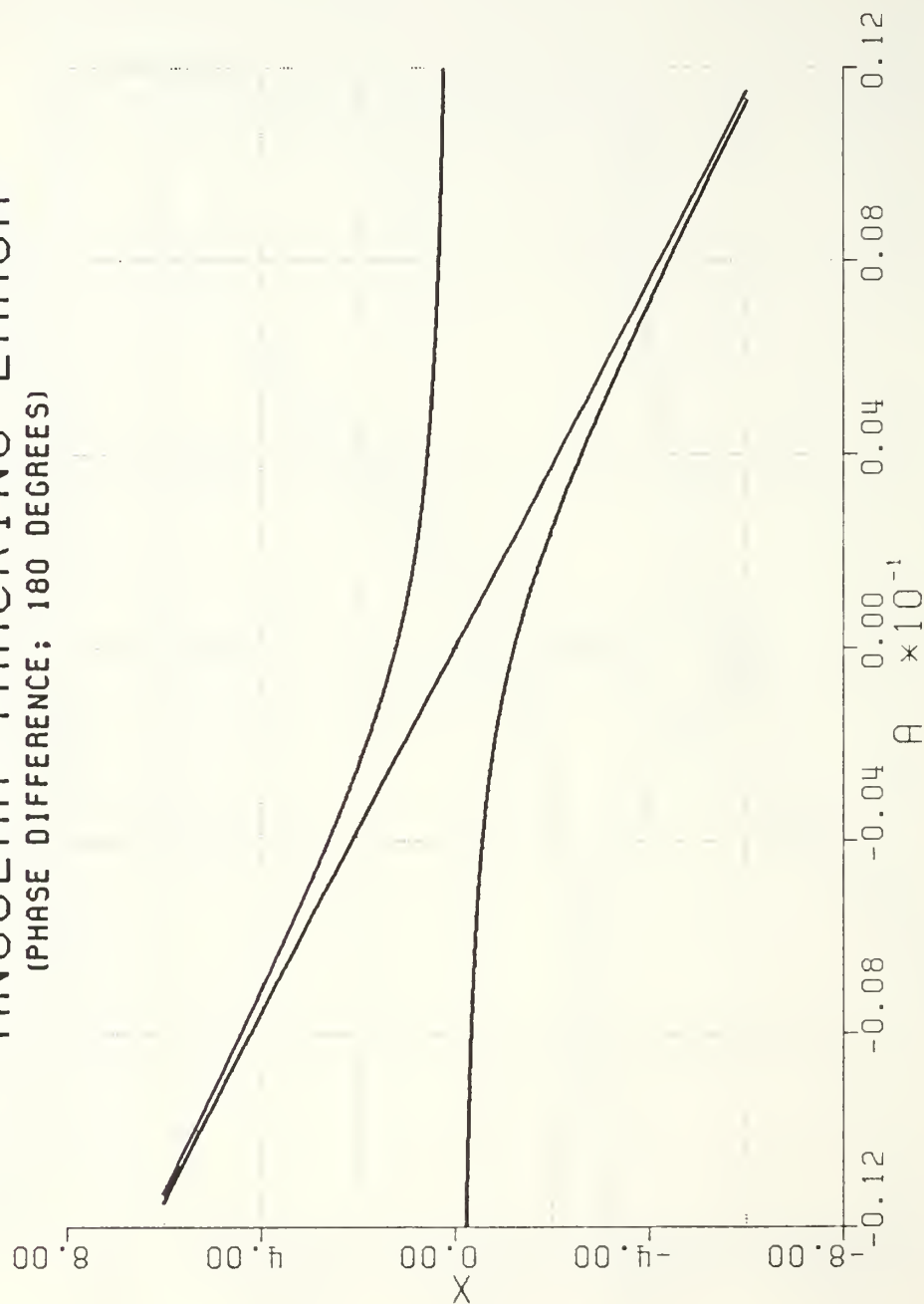


Figure 3.11 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 180 Degrees.

B. CASE 2: $X_C=0.2$ $X_S=0.001$ FOR SEVERAL PHASE DIFFERENCES

Figure 3.12 through 3.22 show X , the normalized tracking error, versus A with the phase difference between the scattered field from the two scatterers as the parameter.

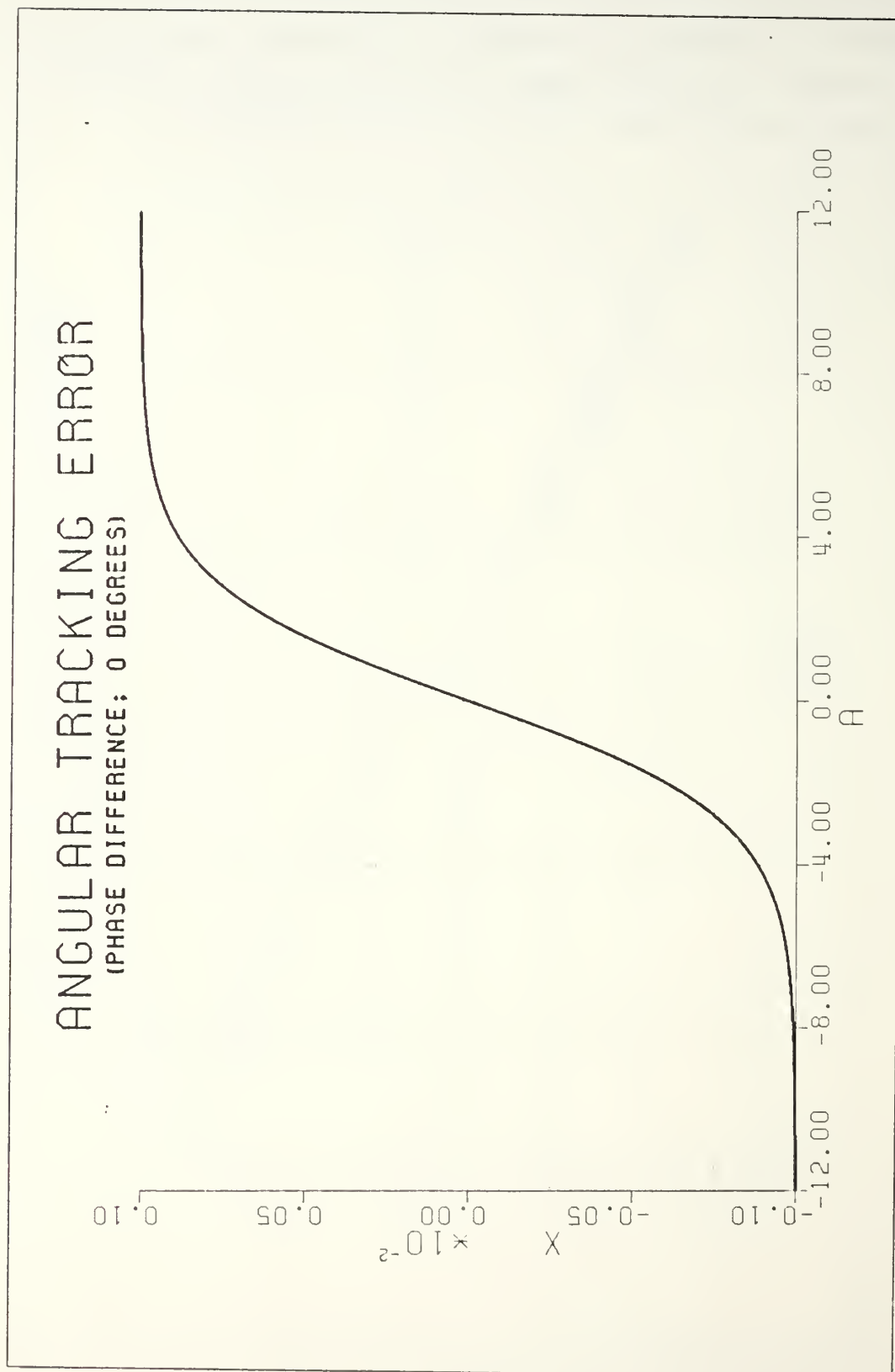


Figure 3.12 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 0 Degrees.

ANGULAR TRACKING ERROR

(PHASE DIFFERENCE; 90 DEGREES)



Figure 3.13 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 100 DEGREES)



Figure 3.14 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 100 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 105 DEGREES)



Figure 3.15 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 105 Degrees..

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 150 DEGREES)

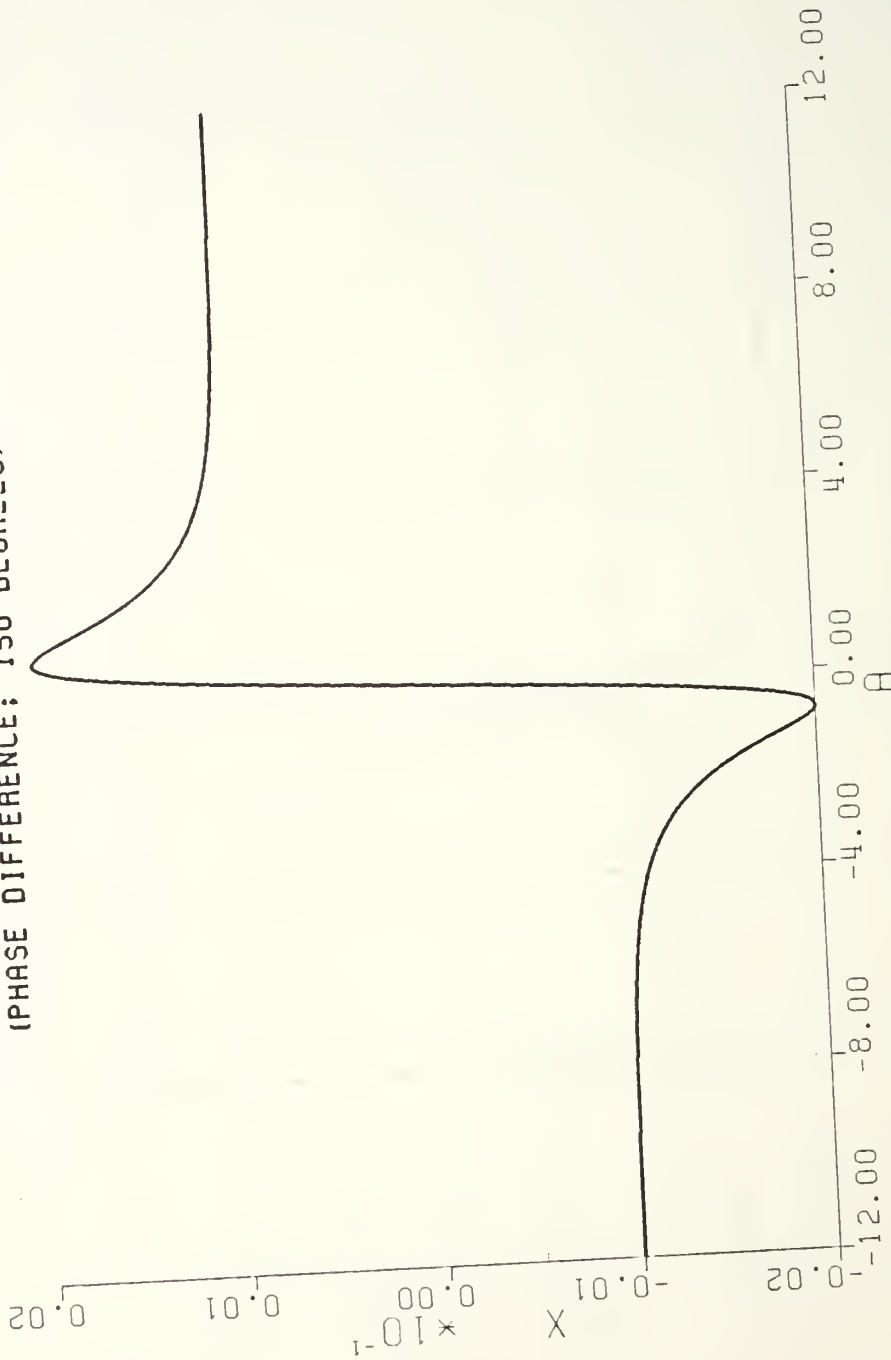


Figure 3.16 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 150 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.88 DEGREES)

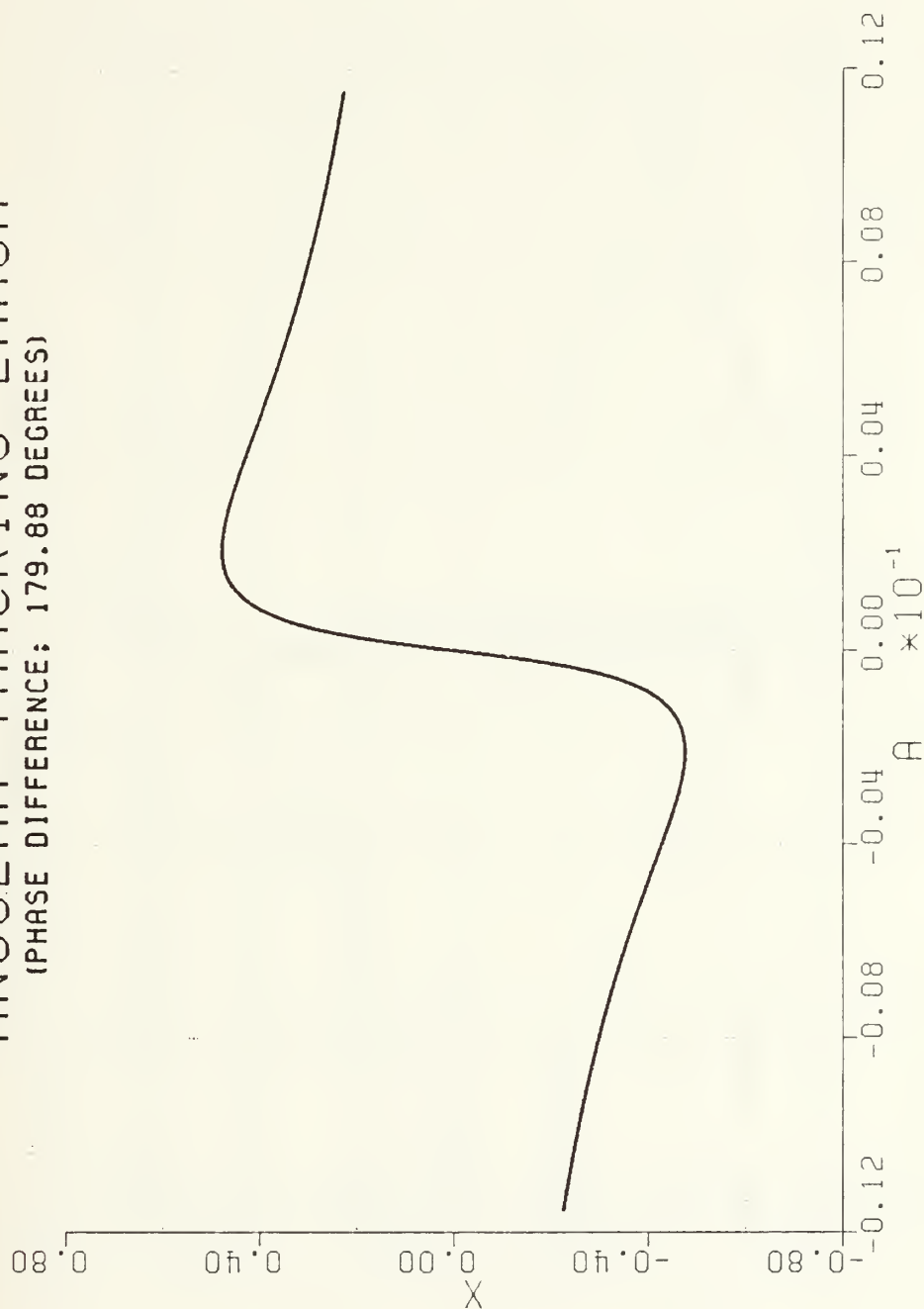


Figure 3.17 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.88 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.90 DEGREES)

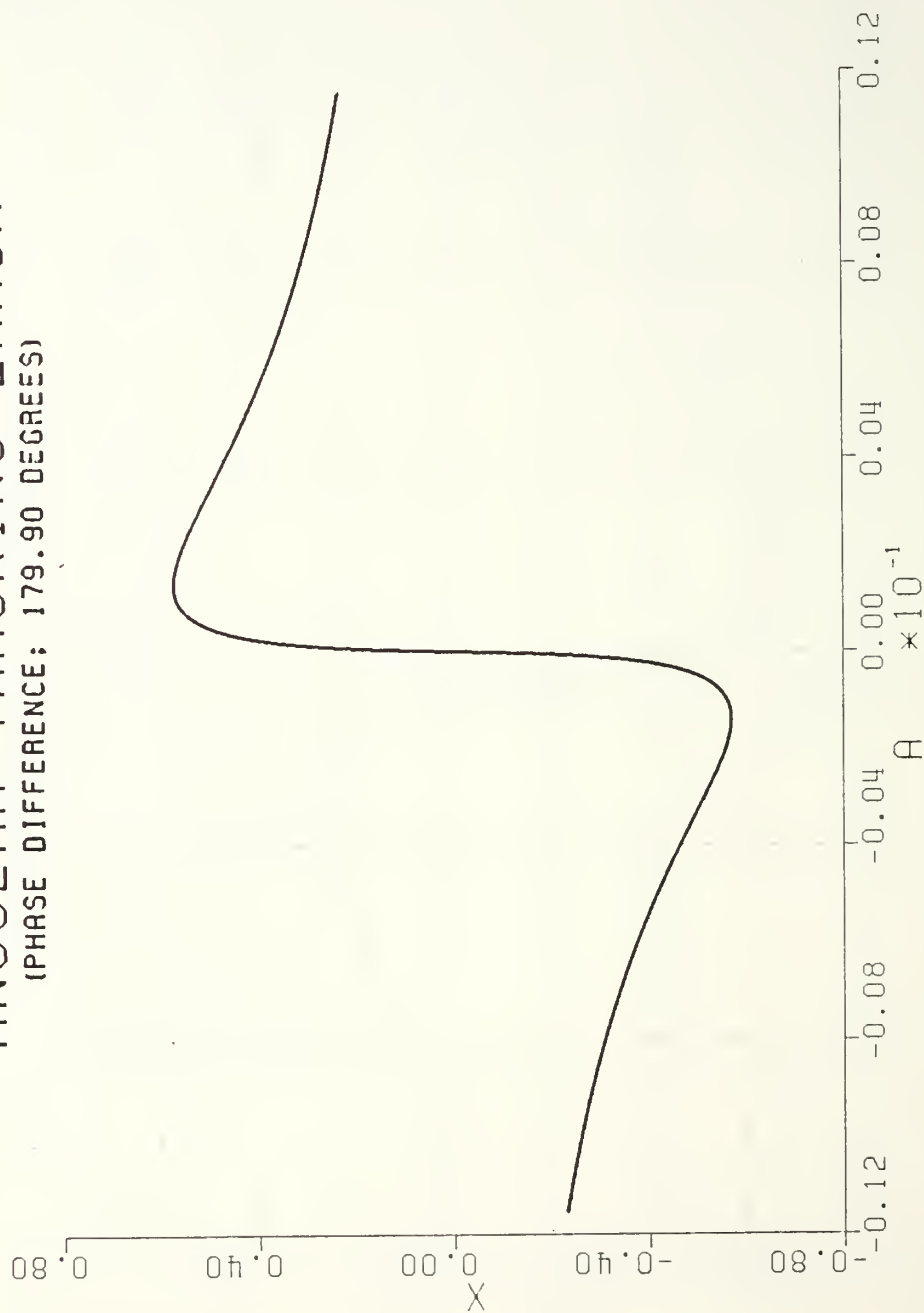


Figure 3.18 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.91 DEGREES)

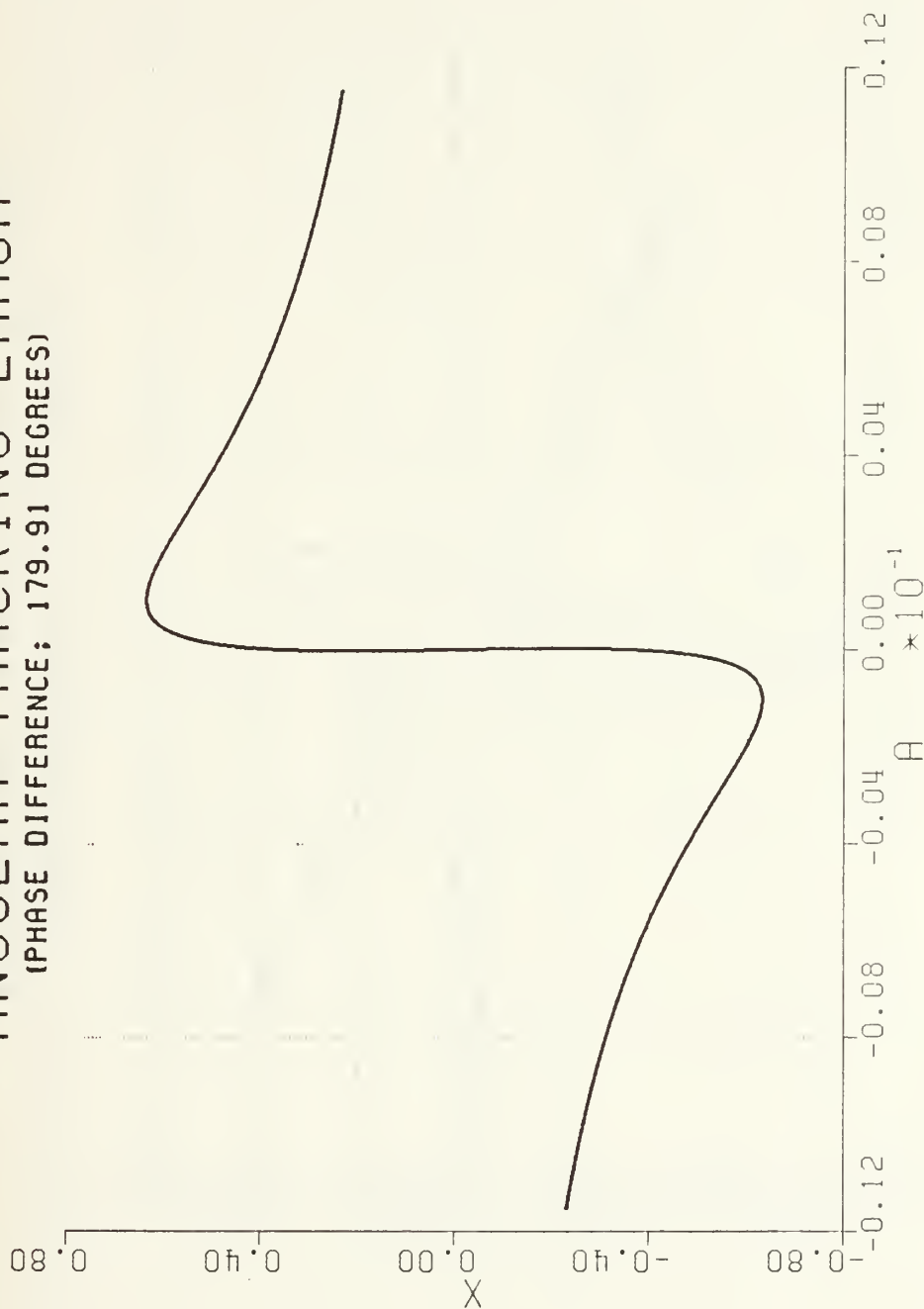


Figure 3.19 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.91 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.95 DEGREES)

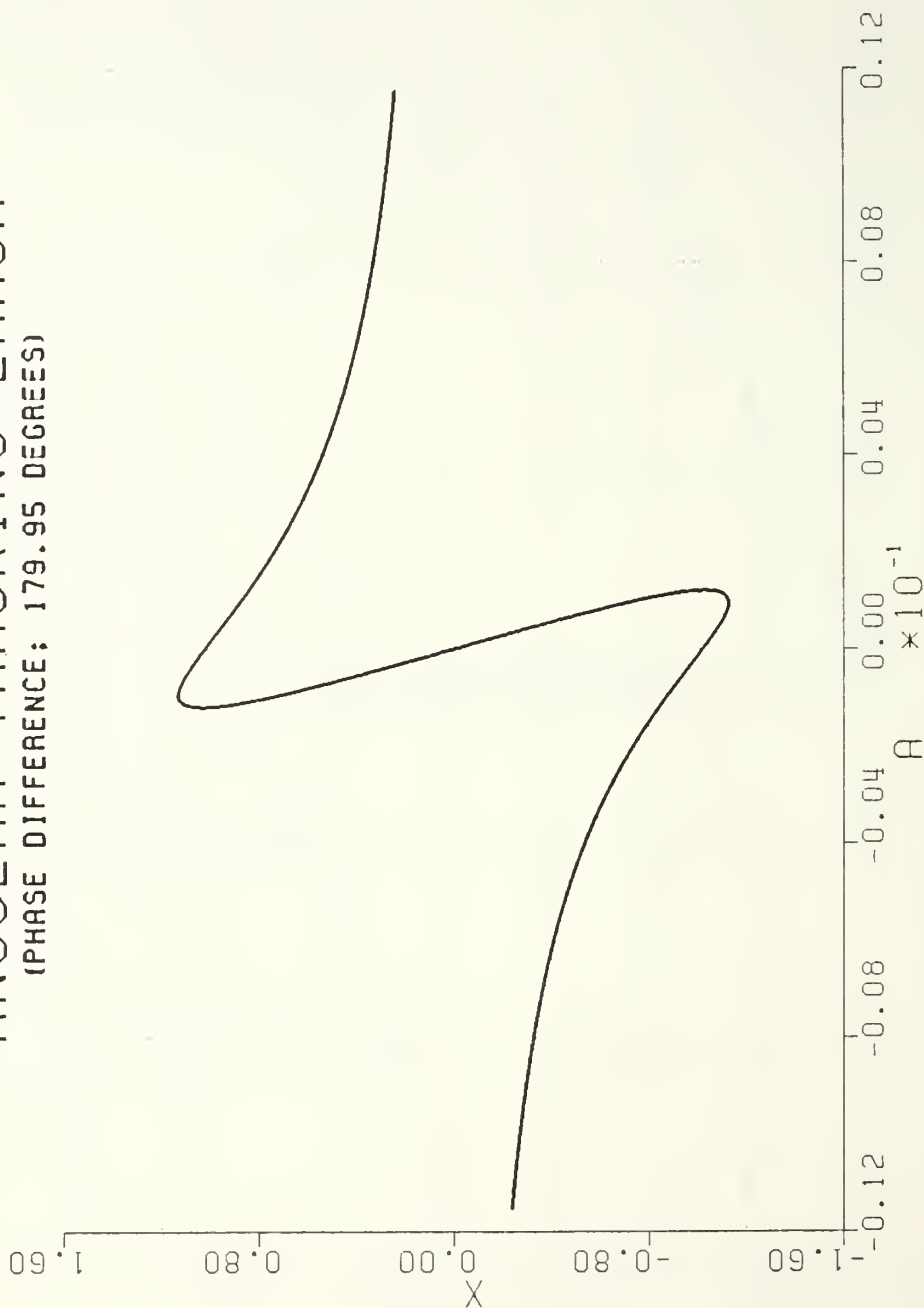


Figure 3.20 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.95 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.99 DEGREES)

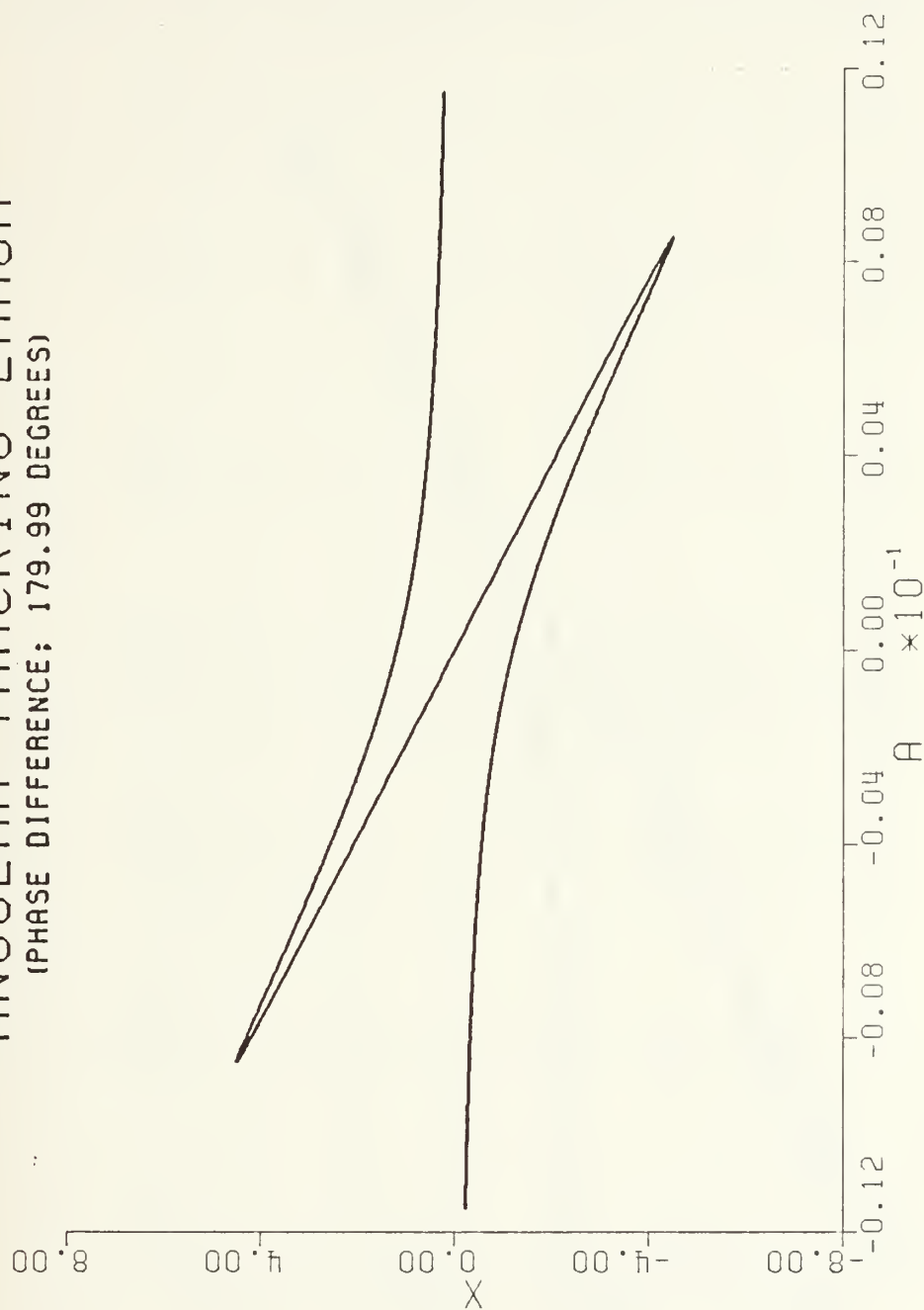


Figure 3.21. Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.99 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 180 DEGREES)

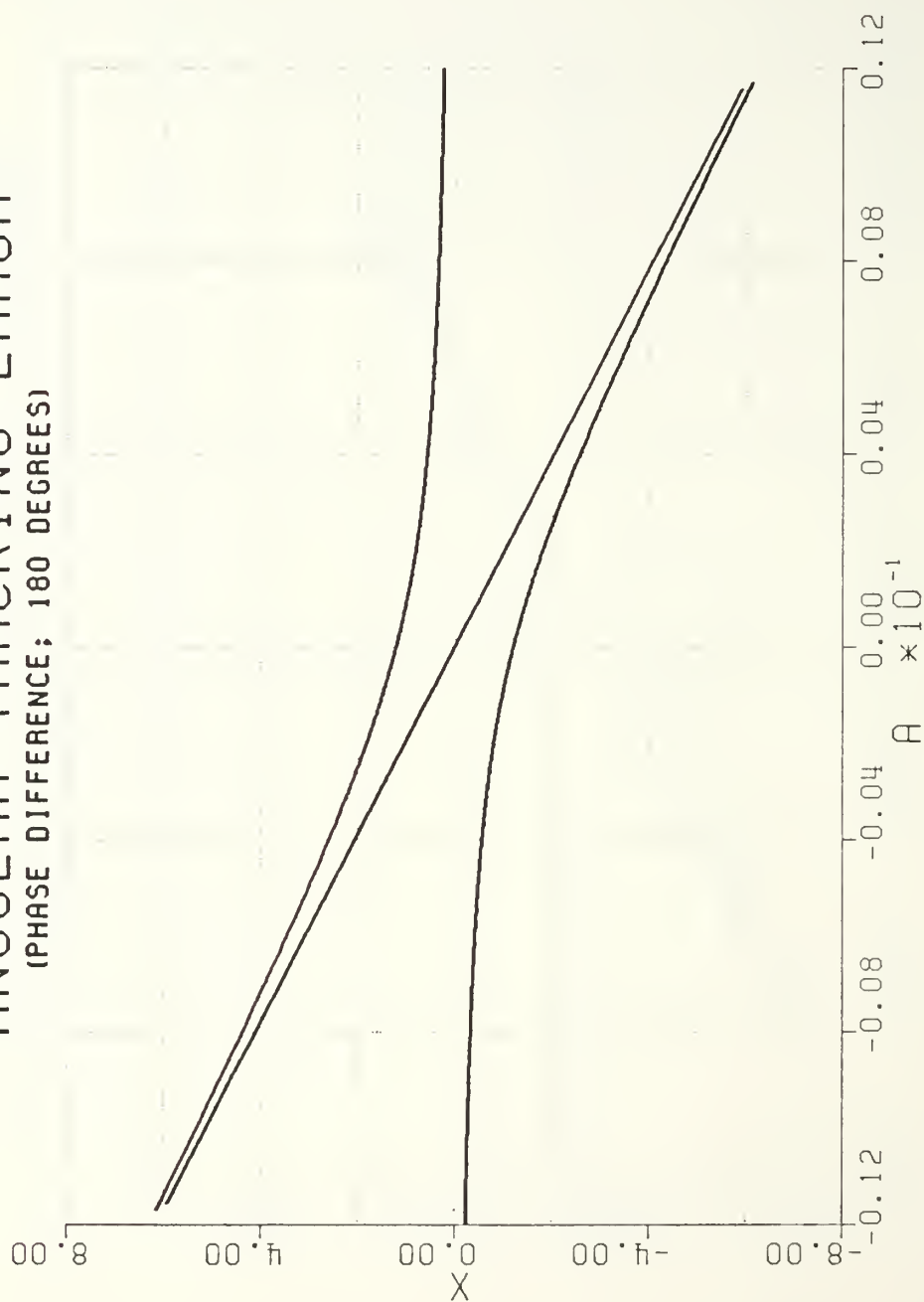


Figure 3.22 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 180 Degrees.

C. CASE 3: $X_C=0.1$, $X_S=0.001$ FOR SEVERAL PHASE DIFFERENCES

Figure 3.23 through 3.33 show X , the normalized tracking error, versus A with the phase difference between the scattered field from the two scatterers as the parameter.

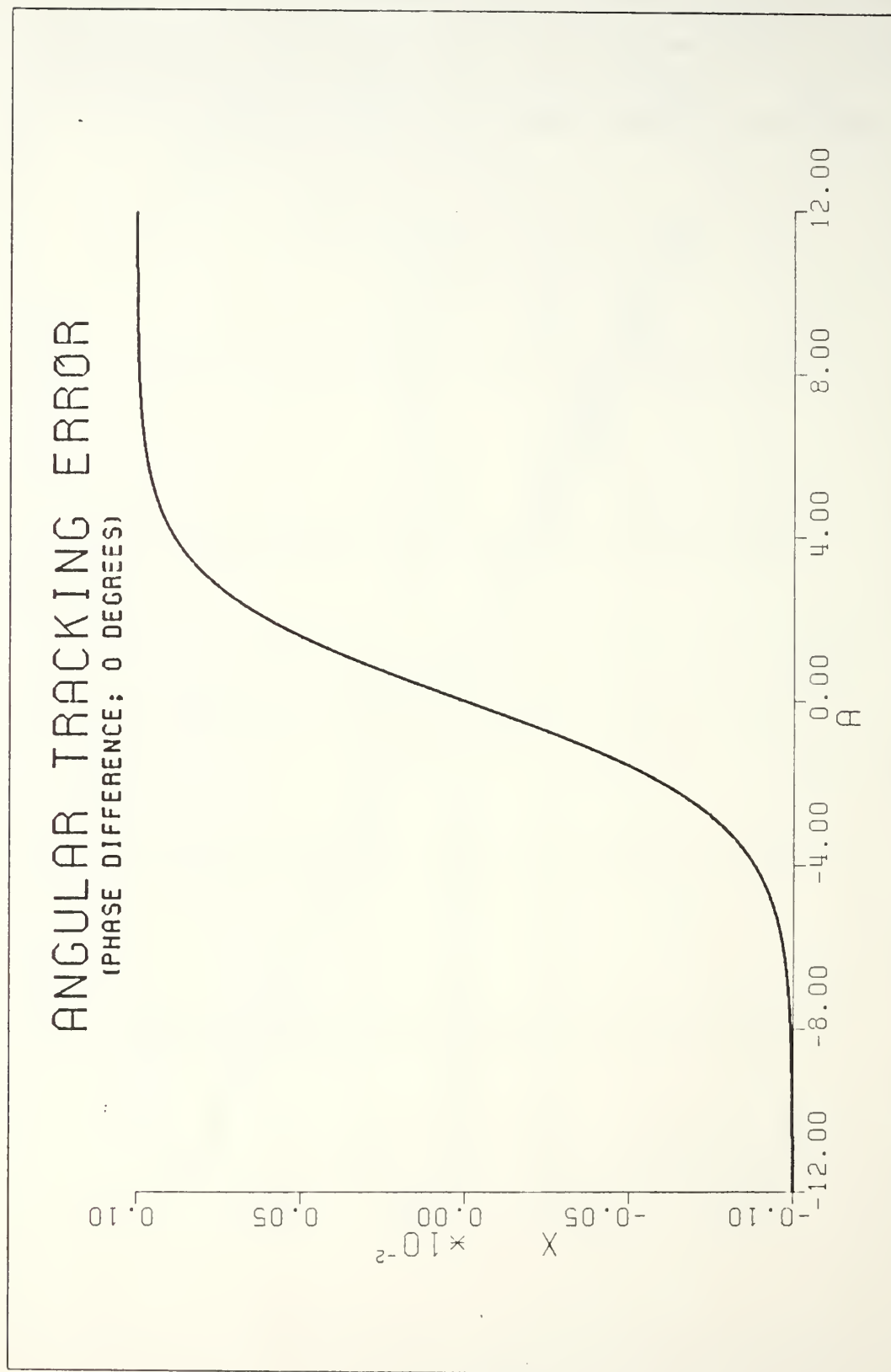


Figure 3.23 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 0 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 90 DEGREES)



Figure 3.24 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 100 DEGREES)



Figure 3.25 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 100 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 105 DEGREES)

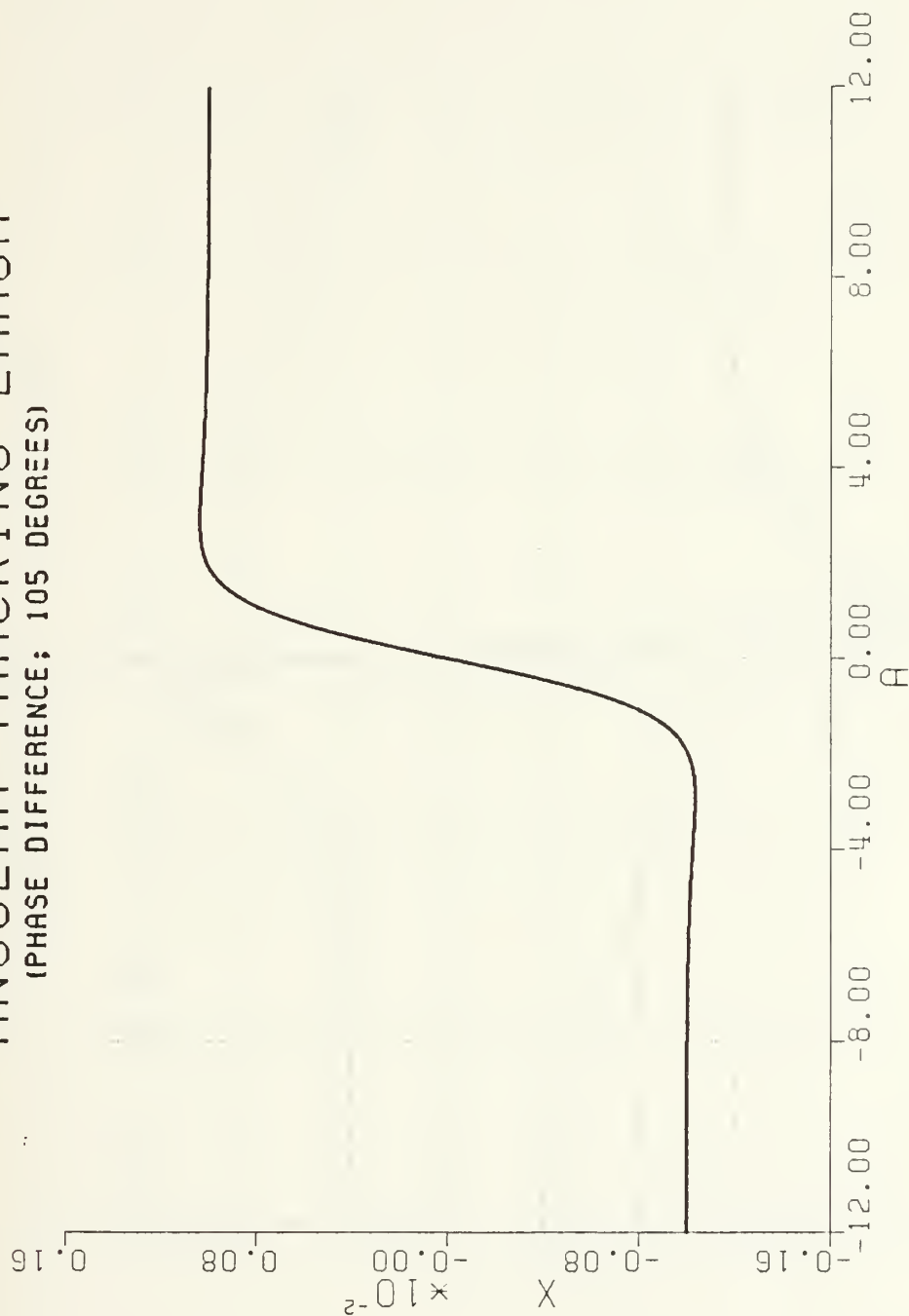


Figure 3.26 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 105 Degrees..

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 150 DEGREES)

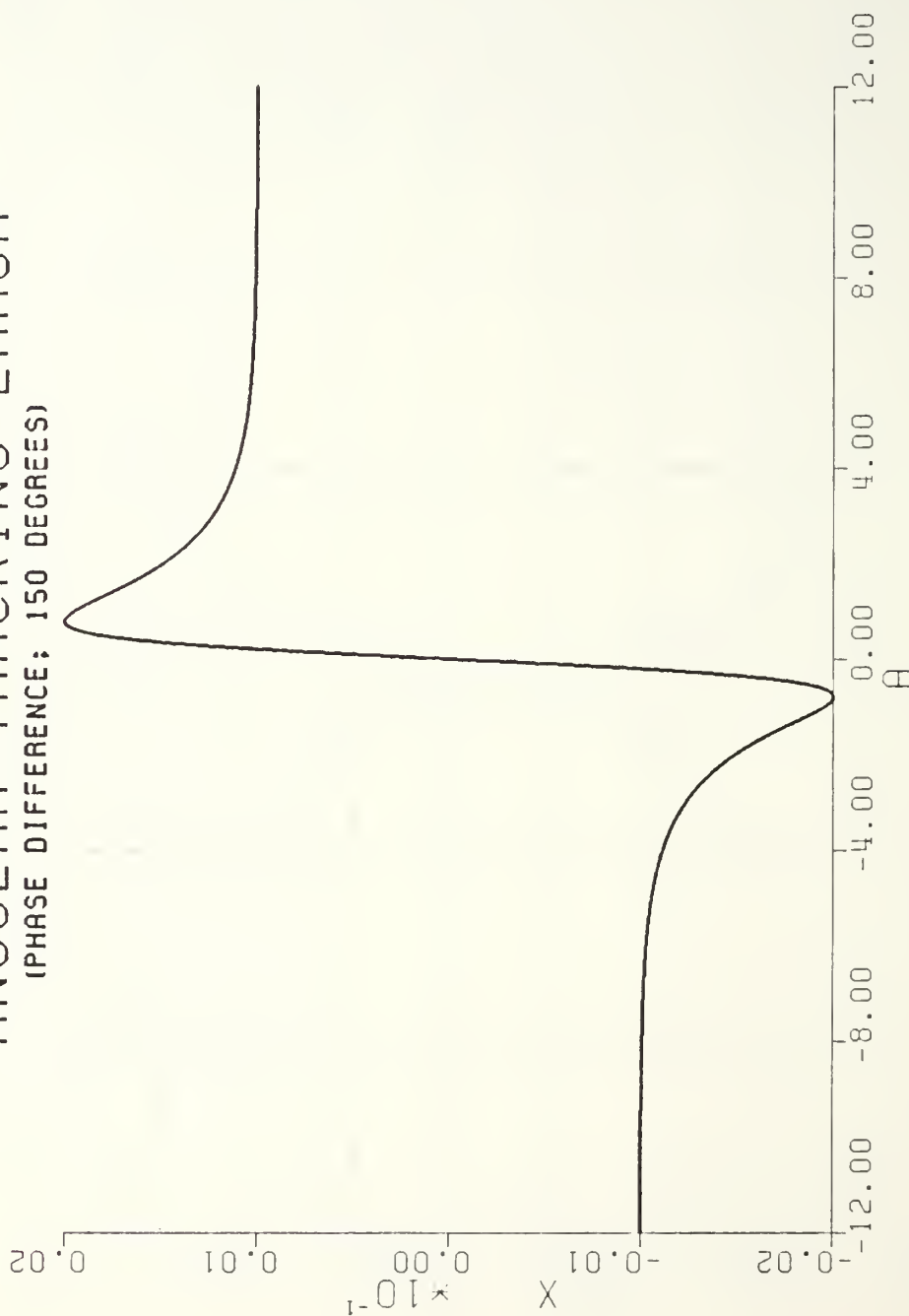


Figure 3.27 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 150 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.88 DEGREES)

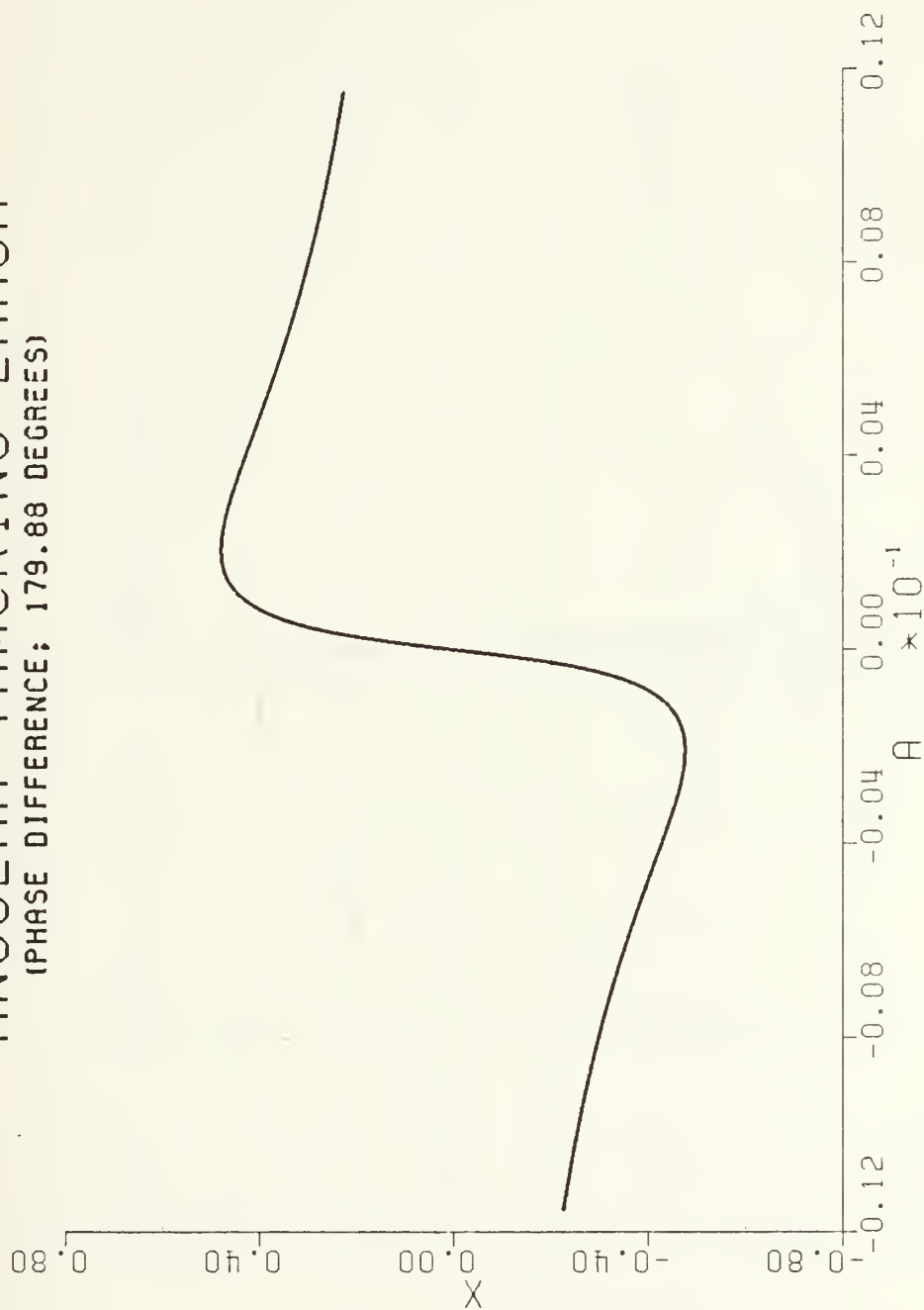


Figure 3.28 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.88 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.90 DEGREES)

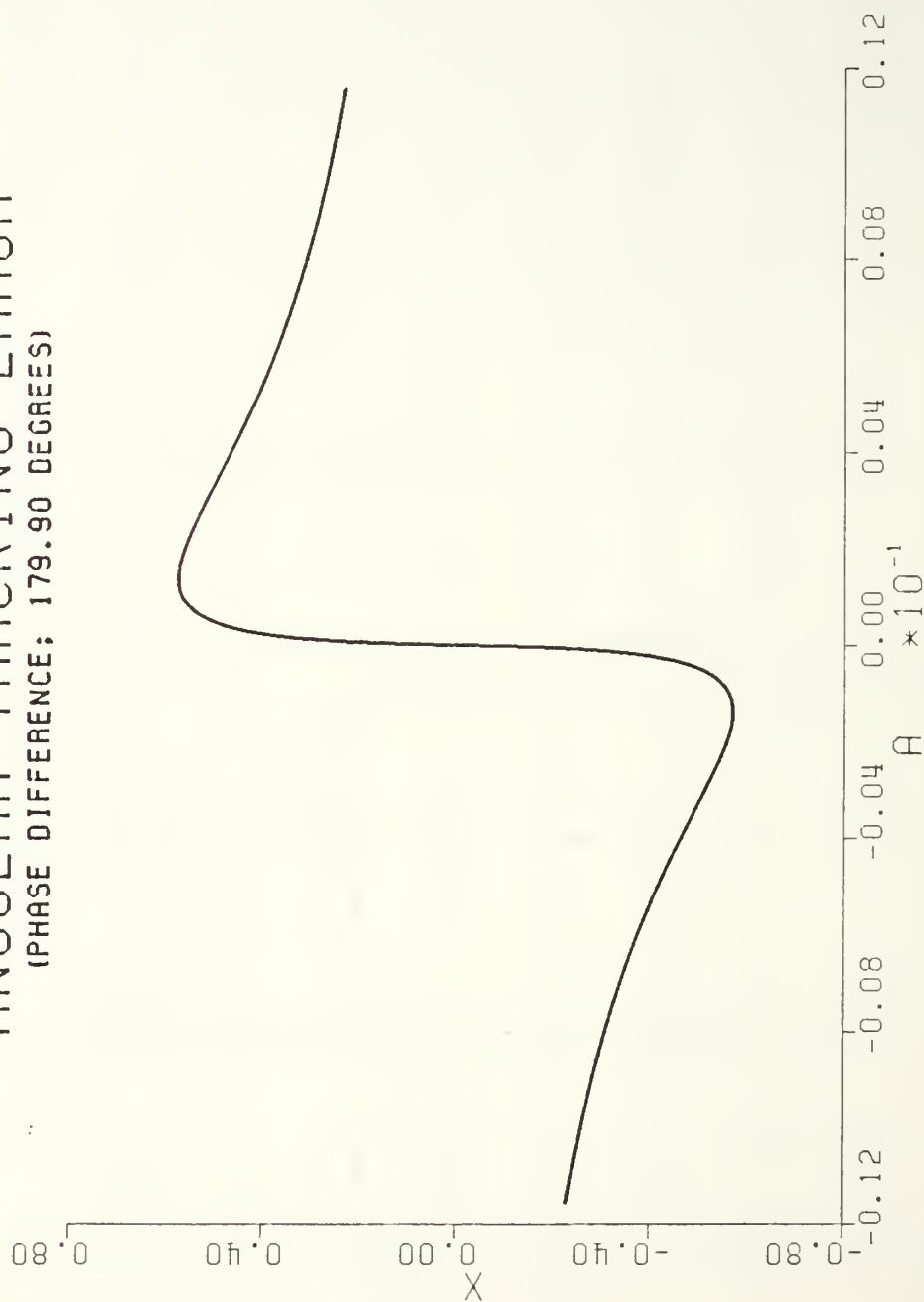


Figure 3.29 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.90 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.91 DEGREES)

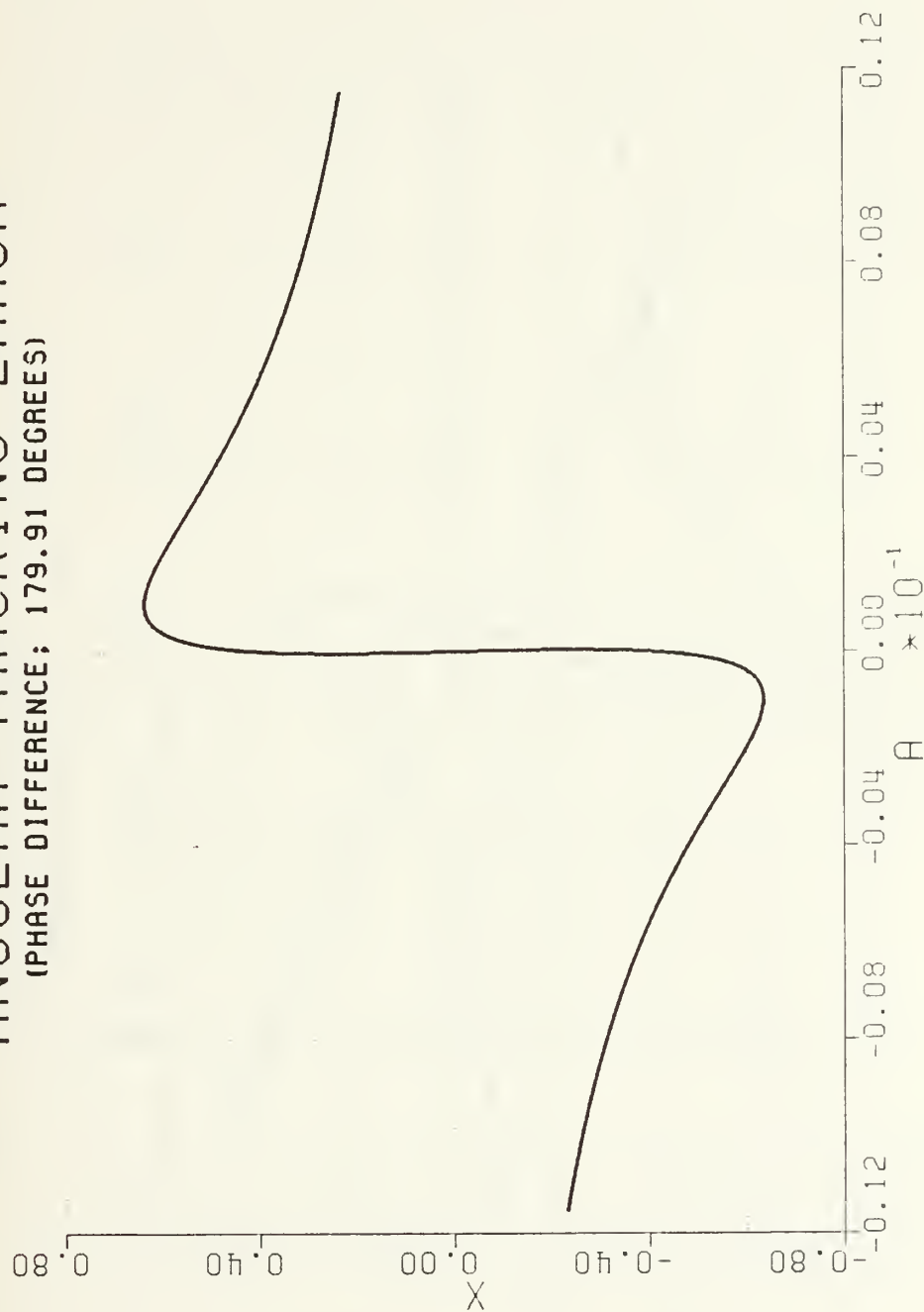


Figure 3.30 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.91 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 179.95 DEGREES)

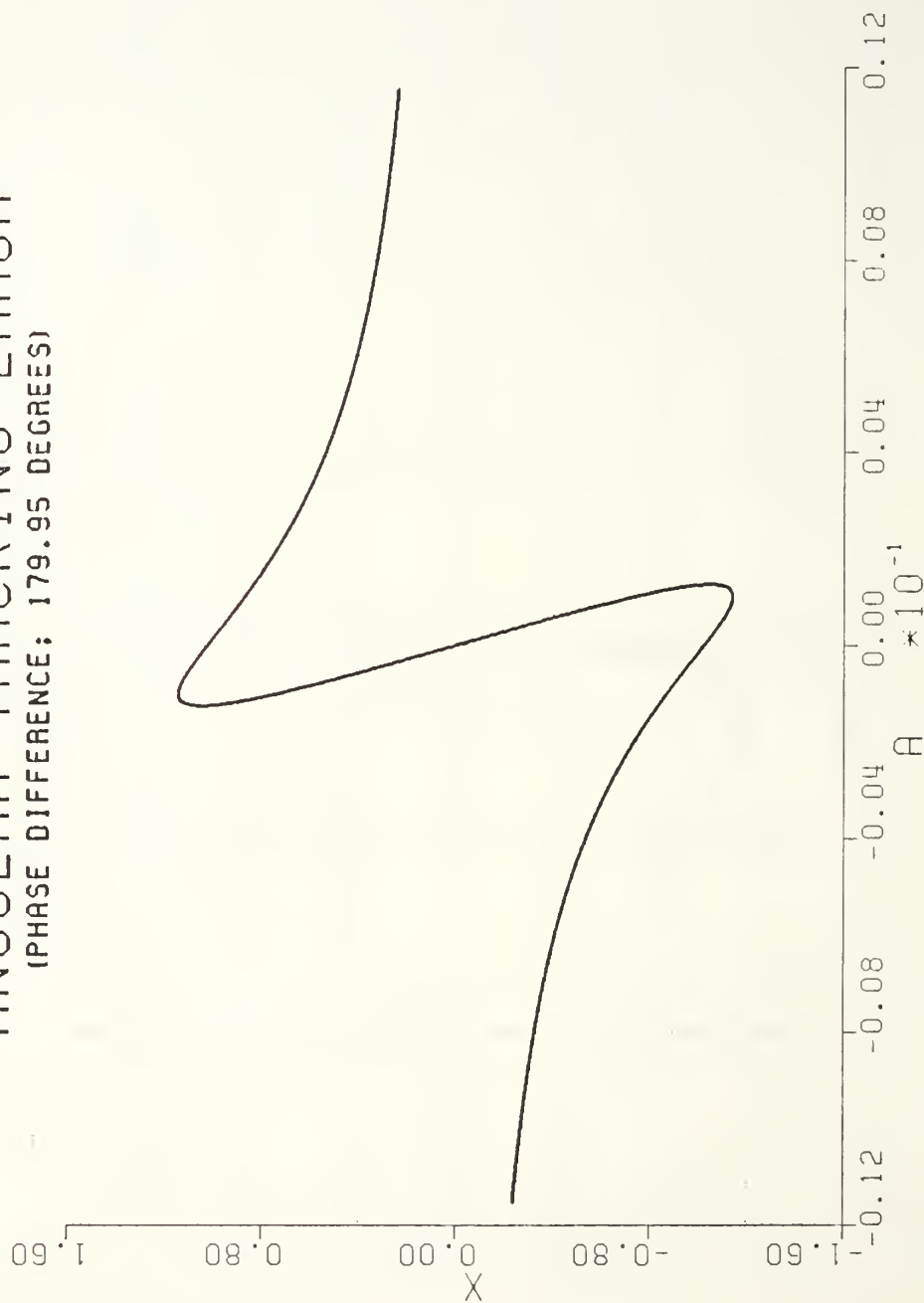


Figure 3.31 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.95 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE: 179.99 DEGREES)

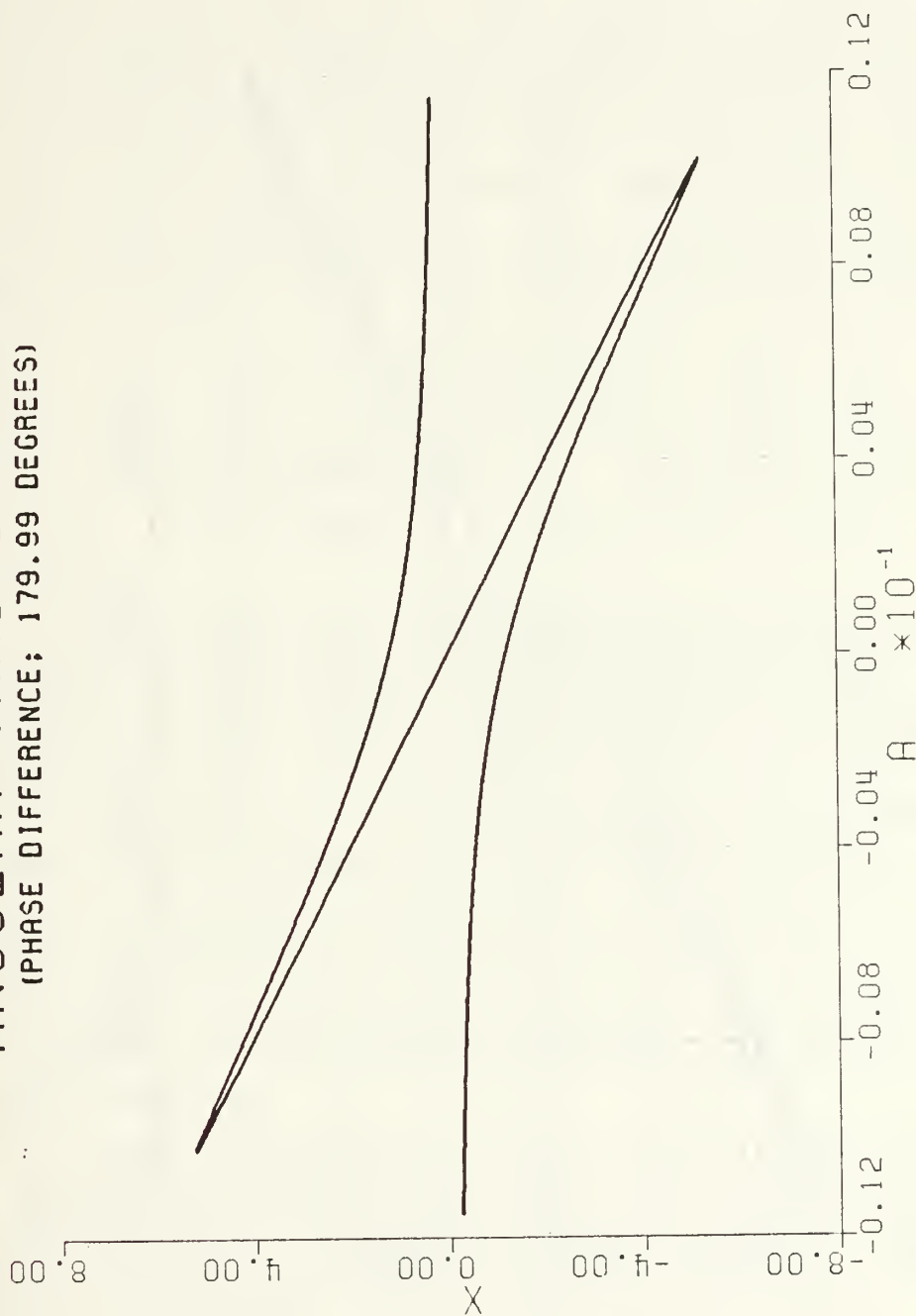


Figure 3.32 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 179.99 Degrees.

ANGULAR TRACKING ERROR (PHASE DIFFERENCE; 180 DEGREES)

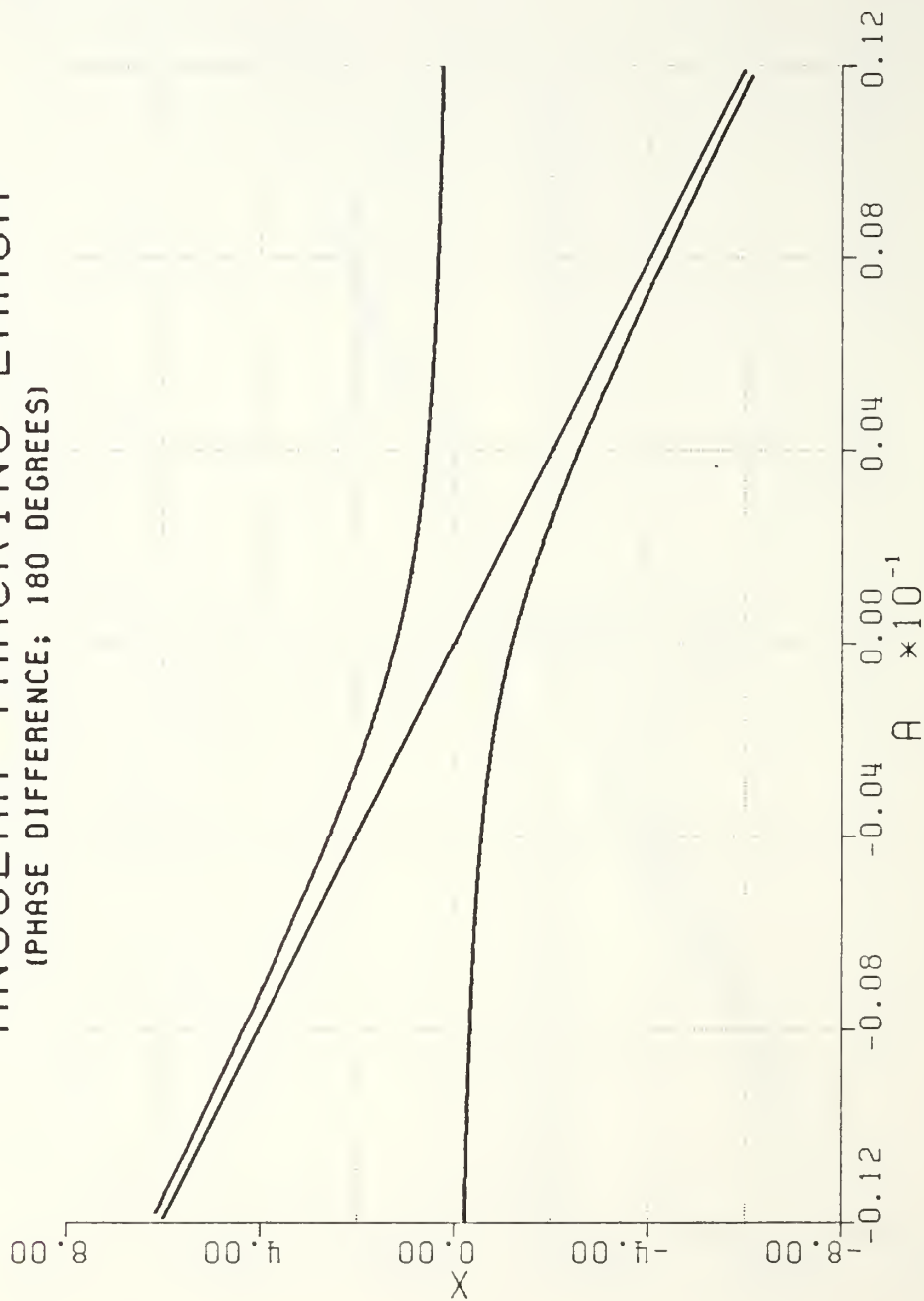


Figure 3.33 Normalized Angular Tracking Error versus Amplitude Ratio for a Target with Two Scattering Centers, Phase Difference between Scattered Fields is 180 Degrees.

IV. CONCLUSION

Meade's analysis of tracking errors under the same assumption made in this thesis predicts an infinite angular error when the scattered fields are equal in strength but 180 out of phase. In this thesis, it is found that there will be no tracking error under this situation. On the other hand, multiple tracking directions are allowed under some circumstances which have never been pointed out before. Since the sidelobes are ignored in the Gaussian radiation pattern, the results in this thesis apply only when the tracking errors are of the same order as the beamwidth, that is, X is not much greater than 1. With this restriction in mind, the above graphical results lead to the following conclusions:

1. When one of the two independent scattering elements of a target scatters more strongly than the other, that is, when the amplitude ratio of two scattered fields is much larger than 1, the system tracks the stronger element.

2. When $\cos(\alpha) > 0$, the inherent angular tracking error is "on target". that is, the tracking system points its axis in a direction within the extremes of the target because the estimated target location from the target geometrical center is in error by less than the angular span of the target. The pointing direction is closer to the stronger scatterer.

3. When the scattered fields from the target elements have approximately equal amplitudes and approach 180 degrees out of phase, the tracking axis may point in one of three different directions without generating an error signal in the tracking system.

4. When $-1 < \cos(\alpha) < 0$, the inherent angular error is bounded. The bound is determined by the phase difference and is given by [Ref. 5].

5. when $-1 < \cos(\alpha) < 0$, the inherent error may contain no turning point or one or two turning points in the region $X > 0$, depending on $\cos(\alpha)$, X_s , and X_c . When there is no turning point, the tracking axis has a unique pointing direction. When one turning point exists, there can be one or three pointing direction. In all cases, when the amplitude ratio of two scattering element is much larger than 1, there is only one pointing direction allowed.

APPENDIX A

COMPUTER PROGRAM FOR COMPUTATION OF ANGULAR TRACKING ERROR

A ; TWO BASED LOGARITHM OF AMPLITUDE RATIO

Q ; PHASE DIFFERENCE IN TWO TARGETS

Xc; NORMALIZED GAUSSIAN BEAM CENTER PHASE

Xs; NORMALIZED CENTER PHASE OF TWO TARGETS

X ; ANGULAR TRACKING ERROR

INTEGER I,J,K,L,M,N

DOUBLE PRECISION A1,X1,FX1,FX2,G1,G2,FX3,Q3

DOUBLE PRECISION X(4001),Y(4001),X5,Y5

REAL X2(4003),Y2(4003)

REAL*4 LMASK1

Q3 = (0.114591456D0*3.141592654D0)/180.0D0

DO 700 I = 1,4001

 A1 = DFLOAT(I)*0.000006D0 -0.012006D0

 X(I) = A1

 G1 = -5.0D0

 G2 = 5.0D0

180 X1 = (G1+G2)/2.0D0

 FX3 = FX1(A1,X1)-FX2(A1,X1)

 IF(FX3.GE.0.0D0) GO TO 160

 G1 = X1

 GO TO 170

160 G2 = X1

170 FX3 = DABS(FX3)

 IF(FX3.GE.1.0E-11) GO TO 180

 Y(I) = X1

700 CONTINUE

DO 850 J = 1,4001

 X5 = X(J)

 Y5 = Y(J)

```

      X(J) = X5*DCOS(Q3)-Y5*DSIN(Q3)
      Y(J) = X5*DSIN(Q3)+Y5*DCOS(Q3)
850 CONTINUE
DO 900 M = 1,4001
      X2(M) = SNGL(X(M))
      Y2(M) = SNGL(Y(M))
900 CONTINUE
      X2(4002) = 0.0
      X2(4003) = 0.0
      Y2(4002) = 0.0
      Y2(4003) = 0.0

DATA LMASK1/Z8888/
CALL PLOTS(0.0,0.0,0.0)
CALL SCALE(X2,6.0,4001,1)
CALL SCALE(Y2,4.0,4001,1)
CALL AXIS(0.5,5.5,'TWO BASED LOGARITHM OF AMPLITUDE RATIO'
*      , -38,6.0,0.0,X2(4002),X2(4003))
CALL AXIS(0.5,5.5,23HANGULAR TRACKING ERROR ,22,4.0,90.0,
*      Y2(4002),Y2(4003))
CALL GRID(0.5,5.5,6,1.0,8,0.5,LMASK1)
CALL NEWPEN(4)
CALL SYMBOL(1.5,9.8,0.2,'ANGULAR TRACKING ERROR',0.0,22)
CALL SYMBOL(1.8,9.6,0.1,'(PHASE DIFFERENCE;179.80 DEGREES)'
*      ,0.0,34)
CALL PLOT(0.5,5.5,-3)
CALL LINE(X2,Y2,4001,1,0,0)
CALL PLOT(0.0,0.0,999)
STOP
END

FUNCTION FX1(P,R)
      DOUBLE PRECISION XC1,Q21,P,R,FX1
      XC1=0.3D0

```

```

Q21=(0.114591406D0*3.141592654D0)/180.0D0
FX1=16.0D0**((P*DSIN(Q21)+R*DCOS(Q21))*XC1)
RETURN
END

FUNCTION FX2(T,S)
  DOUBLE PRECISION XC2,XD,XS,T1,T2,T3,S1,S2,Q1,Q22,T,S,FX2
  Q1 = 3.138101996D0
  Q22 = (0.114591406D0*3.141592654D0)/180.0D0
  XC2=0.3D0
  XD=0.001D0
  XS=XC2*XD
  T1=(T*(DCOS(Q22)+2.0D0*XD*DSIN(Q22))
    +      +S*(2.0D0*XD*DCOS(Q22)-DSIN(Q22))
    +      +2.0D0*XS)*DLOG(2.0D0)
  T2=(T*(DCOS(Q22)+2.0D0*XD*DSIN(Q22))
    +      +S*(2.0D0*XD*DCOS(Q22)-DSIN(Q22))
    +      -2.0D0*XS)*DLOG(2.0D0)
  T3=-DCOS(Q1)
  T1=DCOSH(T1)
  T2=DCOSH(T2)
  IF(T2.EQ.T3) GO TO 100
    S1=T1-T3
    S2=T2-T3
80  FX2=S1/S2
    GO TO 120
100 S1=T1-T3
    S2=1.0E-50
    GO TO 80
120 RETURN
END

```


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2. Howard. D., "Radar Target Angular Scintillation in Tracking and Guidance Systems Based on Echo Signal Phase Front Distortion", Proc. Nat'l Electronics Conf., Vol. 15, pp. 840-849, Oct, 1959.
3. Lee, H-M., "Phase Front Distortion and the Angular Tracking Error of a Phase Comparison Monopulse Tracking Systems", submitted to IEEE Trans Aerospace and Electronic system
4. Lee, H-M., "Analysis of the Inherent Angular Error of Amplitude Comparison Monopulse Tracking", submitted to IEEE trans Aerospace and Electronic Systems.
5. Lee, H-M., Park, D.H., "Inherent Angular Tracking Error of an Amplitude Comparison Monopulse Tracking System", In Preparation.
6. Locke, ed. A.S. Guidance, Van Nostrand, Princeton, pp. 440-442, 1955.
7. Lee, H-M., "One Term Correction to Meade's Analysis of a Two-element Target", submitted to IEEE Trans on Aerospace and Electronic Systems.

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